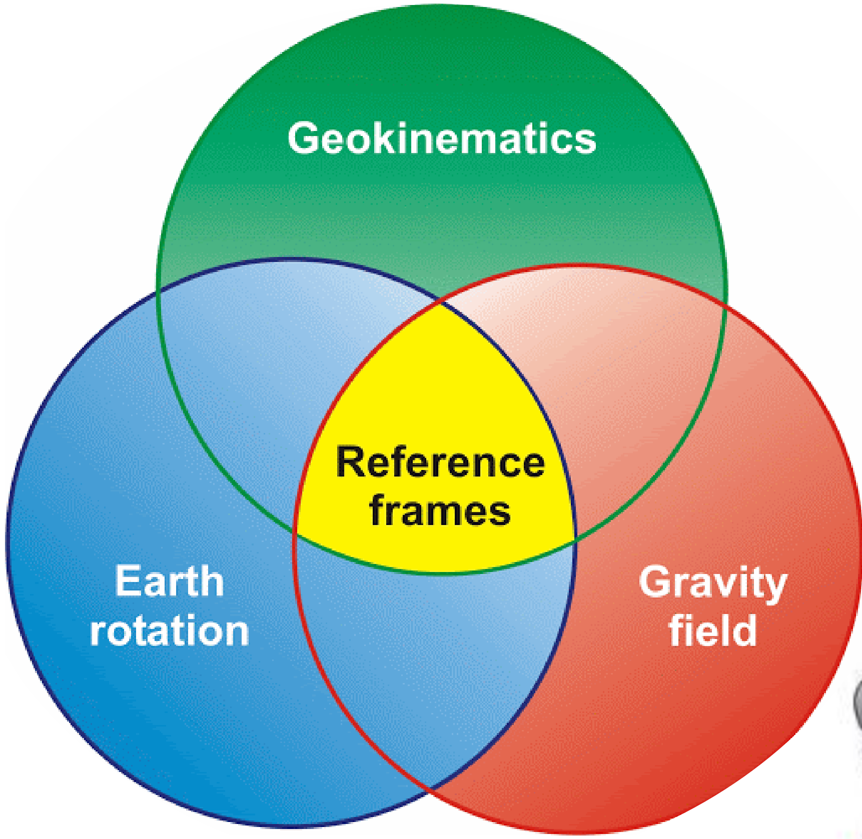
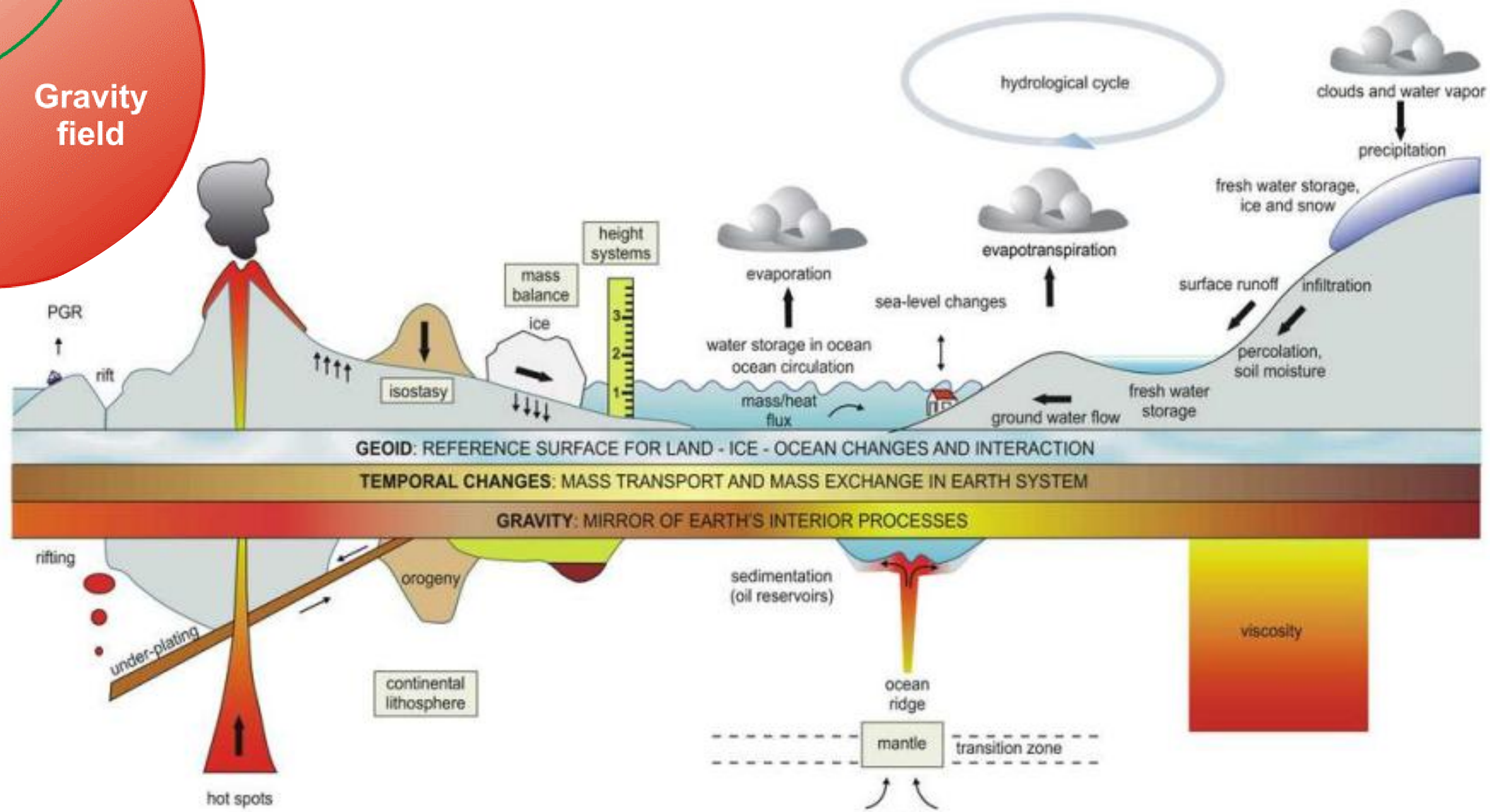
# GEODESIA

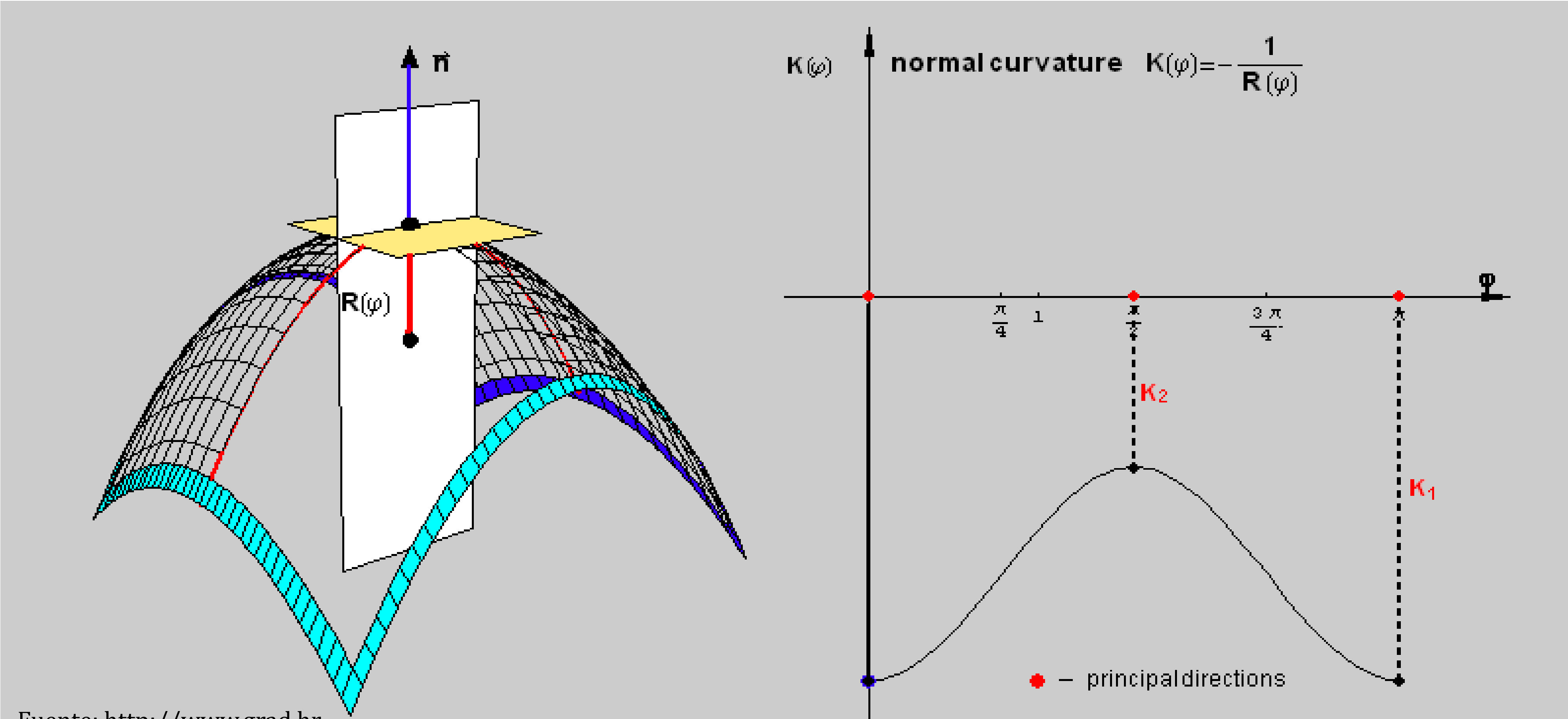



Plag, H. P., Rothacher, M., Pearlman, M., Neilan, R., & Ma, C. (2009). The global geodetic observing system. In *Advances in Geosciences: Volume 13: Solid Earth (SE)* (pp. 105-127).

# Geodesia

## Curvatura y Radio de Curvatura [6,7,9]:

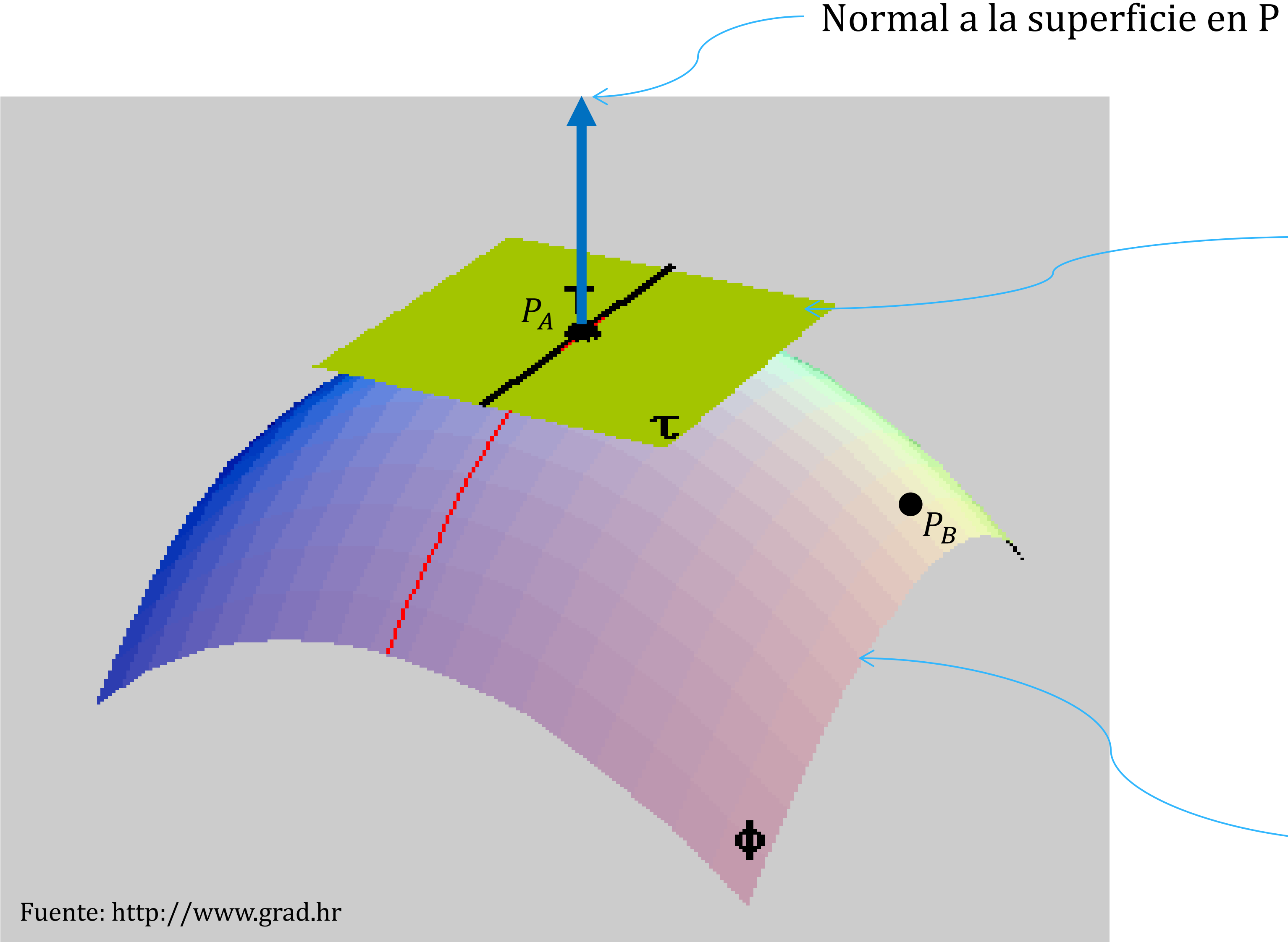



Fuente: http://www.grad.hr


(6) Krakiwsky, Edward J., and Donald B. Thomson (1974). *Geodetic position computations*.. Department of Surveying Engineering, University of New Brunswick.
(7) Rapp, Richard H. (1991). *Geometric geodesy part I*. Department of Geodetic Science and Surveying, Ohio State University.
(9) Jekeli, C. (2006). *Geometric reference systems in geodesy*. Division of Geodetic Science, School of Earth Sciences. Ohio State University.

# Geodesia

**Curvas[6,7,9]:**



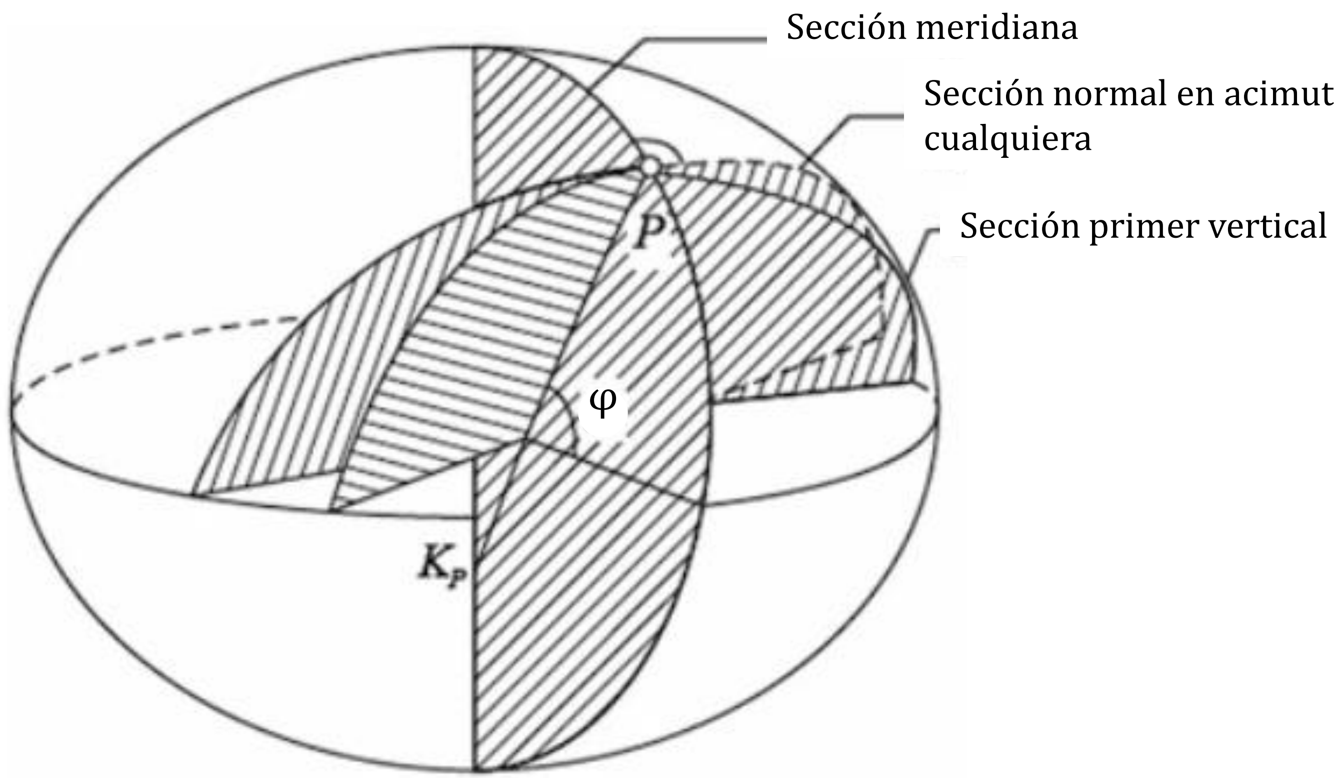

Curva de sección normal: es una curva plana creada por la intersección de un plano que contiene la normal a la superficie (un plano de sección normal) con la superficie.


(6) Krakiwsky, Edward J., and Donald B. Thomson (1974). *Geodetic position computations*.. Department of Surveying Engineering, University of New Brunswick.
(7) Rapp, Richard H. (1991). *Geometric geodesy part I*. Department of Geodetic Science and Surveying, Ohio State University.
(9) Jekeli, C. (2006). *Geometric reference systems in geodesy*. Division of Geodetic Science, School of Earth Sciences. Ohio State University.

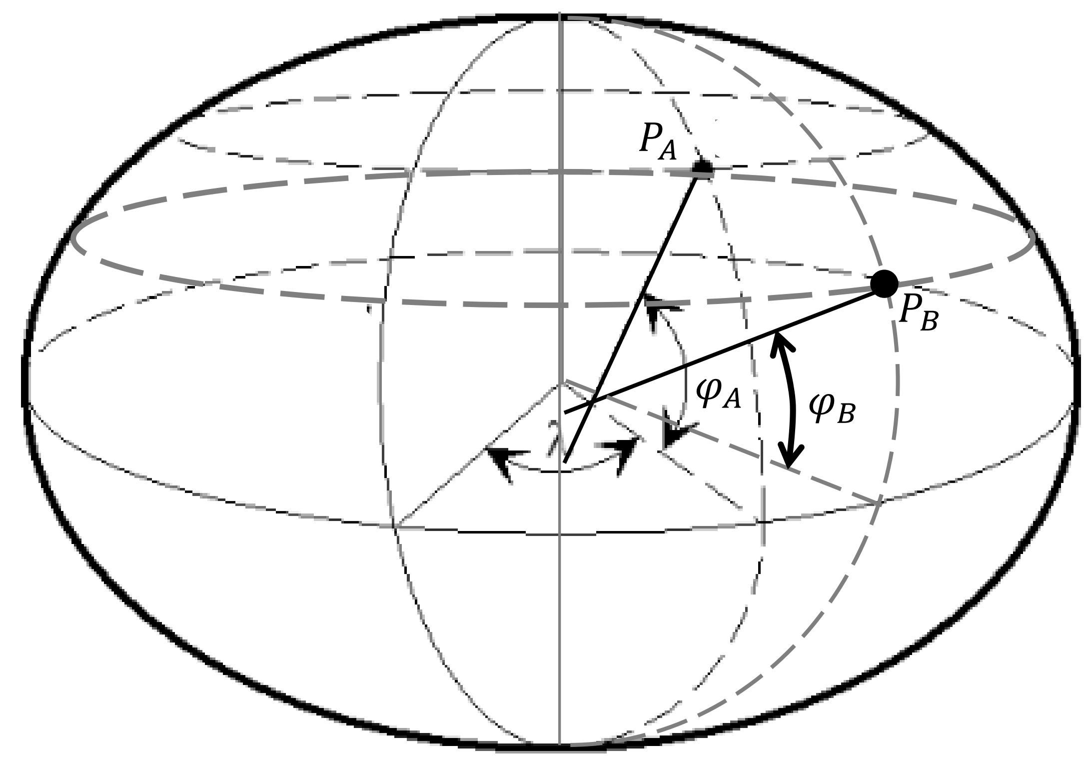
# Geodesia

**Secciones en el elipsoide(7,9,10):**

Sección Normal (definición matemática): curva formada por la intersección de un plano que contiene a la normal a la superficie del elipsoide en un punto dado.




Fuente: Lu Z., Qu Y., Qiao S. (2014)





(7) Rapp, Richard H. (1991). *Geometric geodesy part I*. Department of Geodetic Science and Surveying, Ohio State University.
(9) Jekeli, C. (2006). *Geometric reference systems in geodesy*. Division of Geodetic Science, School of Earth Sciences. Ohio State University.
(10) Jordan, W. (1962). *Handbook of Geodesy* (Vol. 3). Corps of Engineers, United States Army, Army Map Service

# Geodesia

## Secciones en el elipsoide[7,9,10]:

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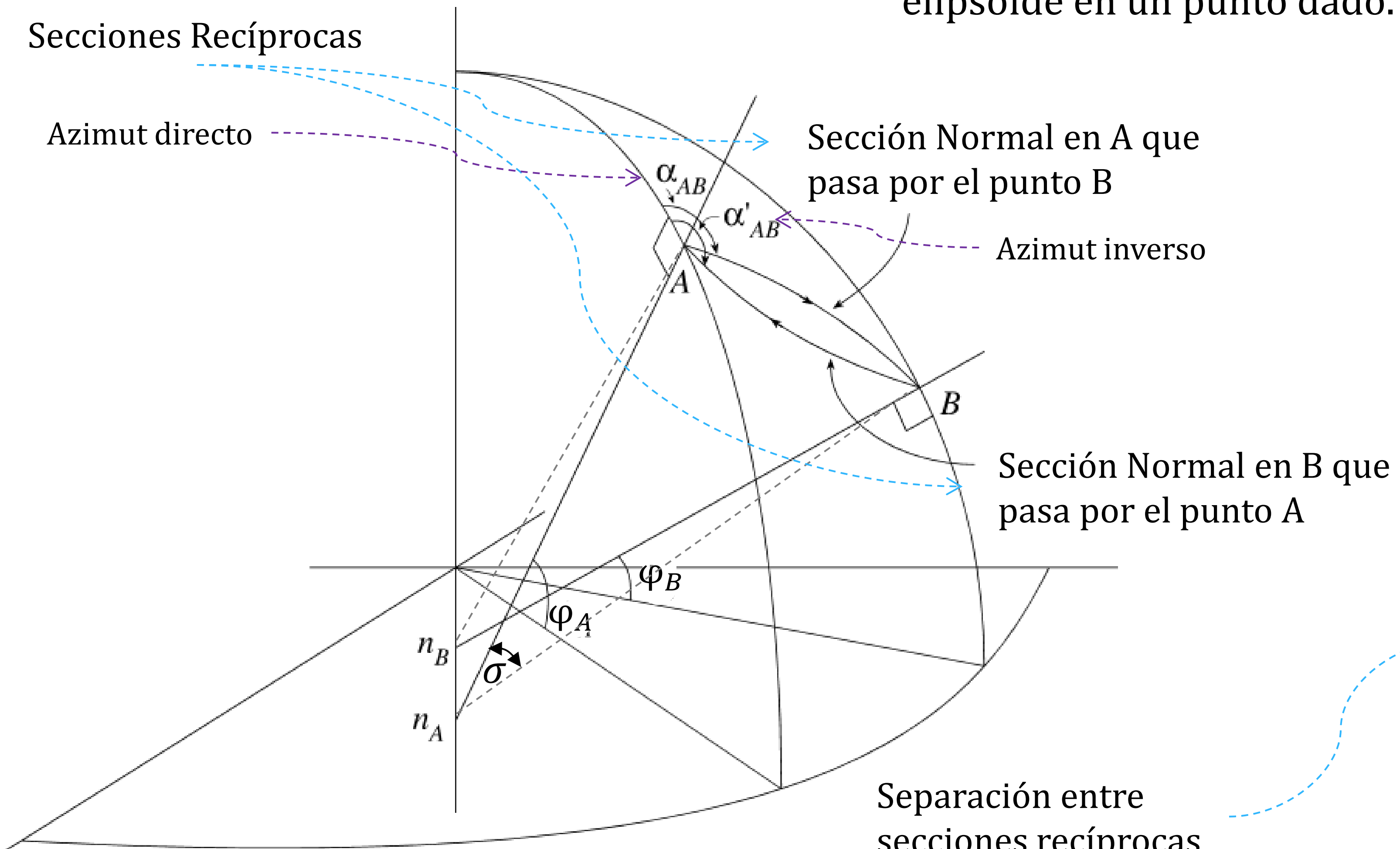
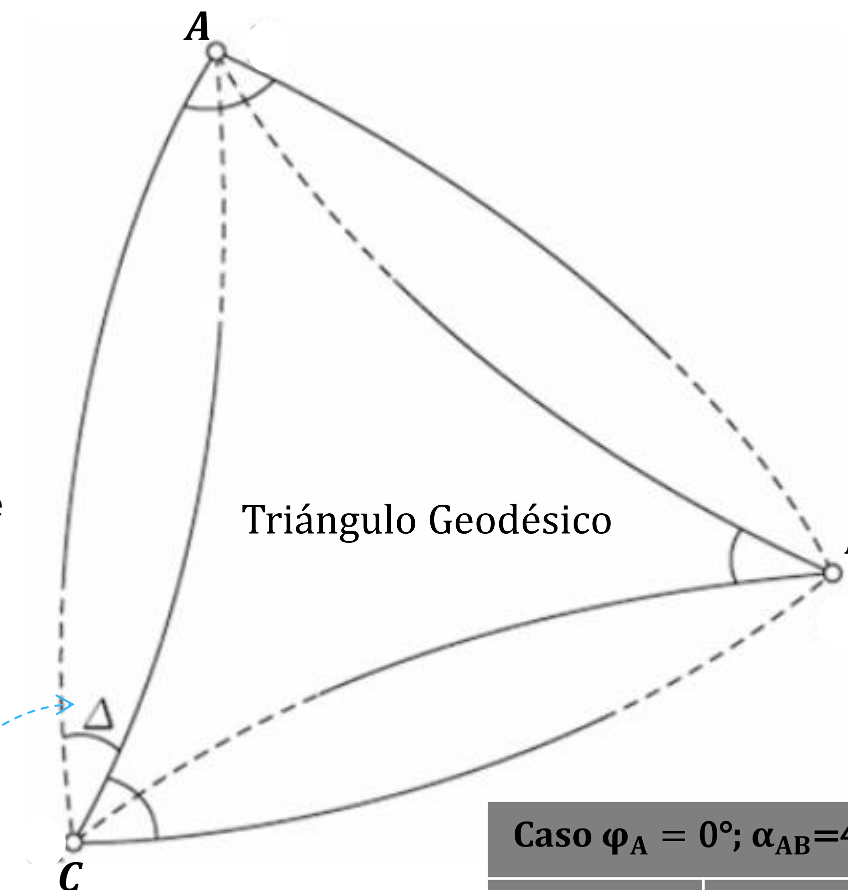



Fuente: Jekeli, C. (2006)




Separación entre secciones recíprocas

$$\Delta = \alpha_{AB} - \alpha'_{AB} = \frac{e^2}{4}\left(\frac{S_{AB}}{N_A}\right)^2 sen2\alpha_{AB}\,cos^2\varphi_m$$

| Caso $\varphi_A = 0°$; $\alpha_{AB}=45°$ | |
|---|---|
| S (km) | Δ (”) |
| 200 | 0.399 |
| 100 | 0.085 |
| 50 | 0.021 |


**(7) Rapp, Richard H. (1991).** ***Geometric geodesy part I*****. Department of Geodetic Science and Surveying, Ohio State University.**
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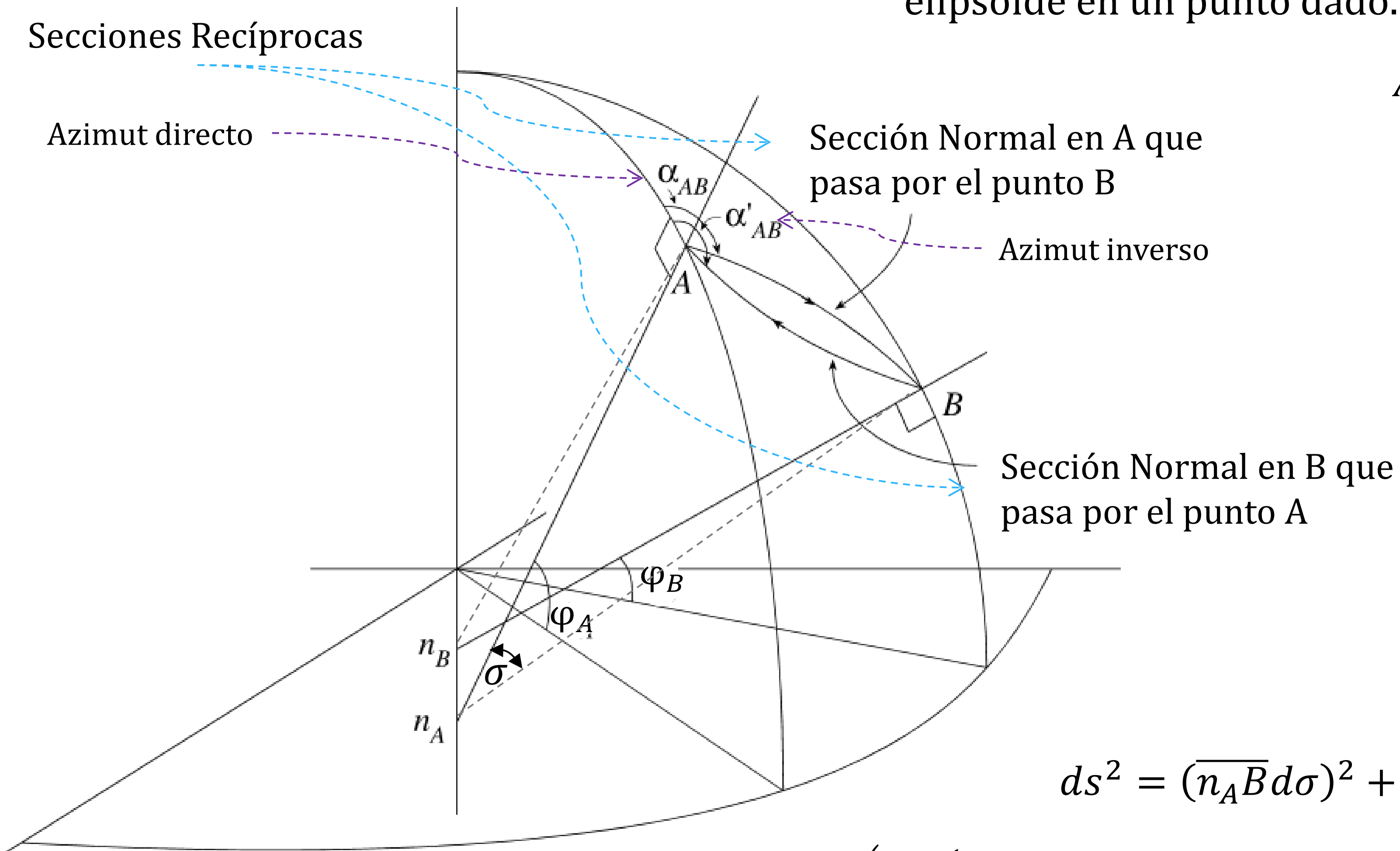
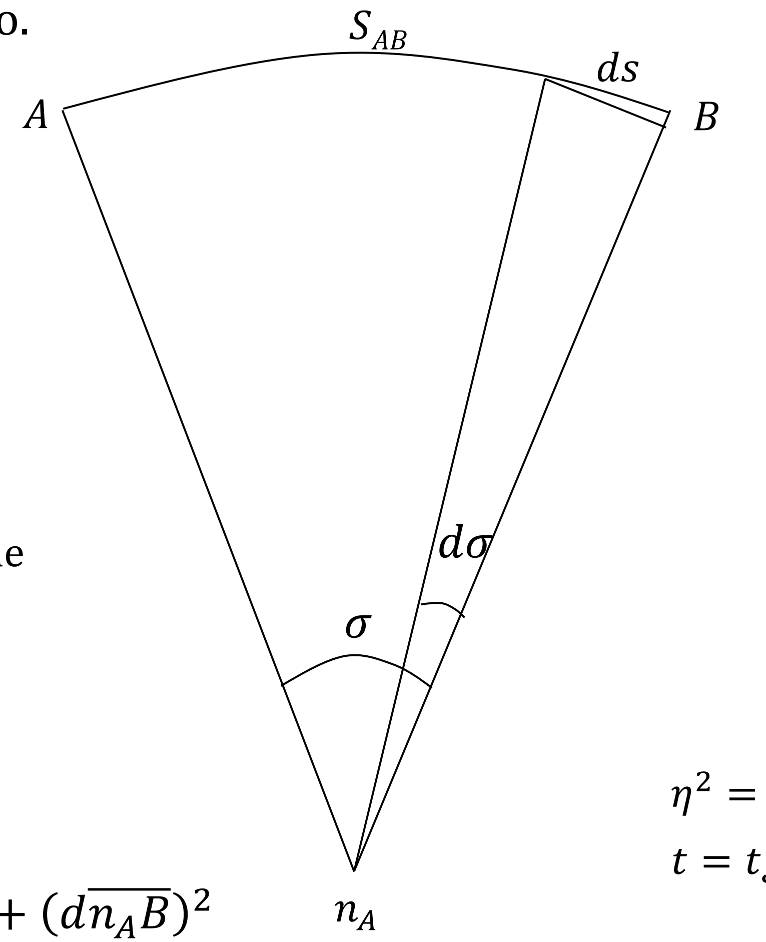
## Secciones en el elipsoide[7,9,10]:

Sección Normal (definición matemática): curva formada por la intersección de un plano que contiene a la normal a la superficie del elipsoide en un punto dado.




Fuente: Jekeli, C. (2006)




$$\eta^2 = e'^2 cos^2\varphi$$

$$t = tg\varphi$$

$$ds^2 = (\overline{n_A B}\, d\sigma)^2 + (d\overline{n_A B})^2$$

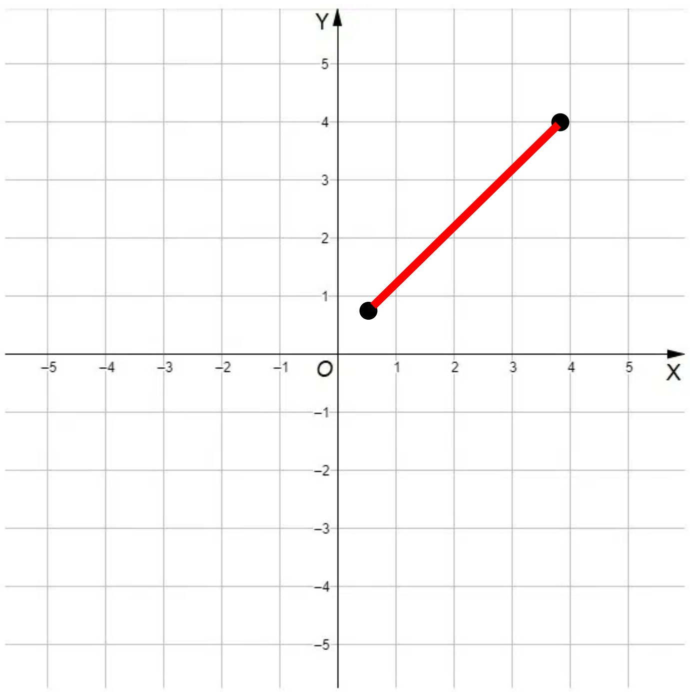
$$S_{AB} = N_A\sigma\left(1 + \frac{1}{6}\sigma^2\eta_A{}^2\, cos^2\alpha_{AB}(\eta_A{}^2\, cos^2\alpha_{AB} - 1) + \frac{1}{8}t_A\eta_A{}^2\, cos\alpha_{AB}(1 - 3\,\eta_A{}^2\, cos^2\alpha_{AB})\sigma^3\right)$$


(7) Rapp, Richard H. (1991). *Geometric geodesy part I*. Department of Geodetic Science and Surveying, Ohio State University.
(9) Jekeli, C. (2006). *Geometric reference systems in geodesy*. Division of Geodetic Science, School of Earth Sciences. Ohio State University.
(10) Jordan, W. (1962). *Handbook of Geodesy* (Vol. 3). Corps of Engineers, United States Army, Army Map Service

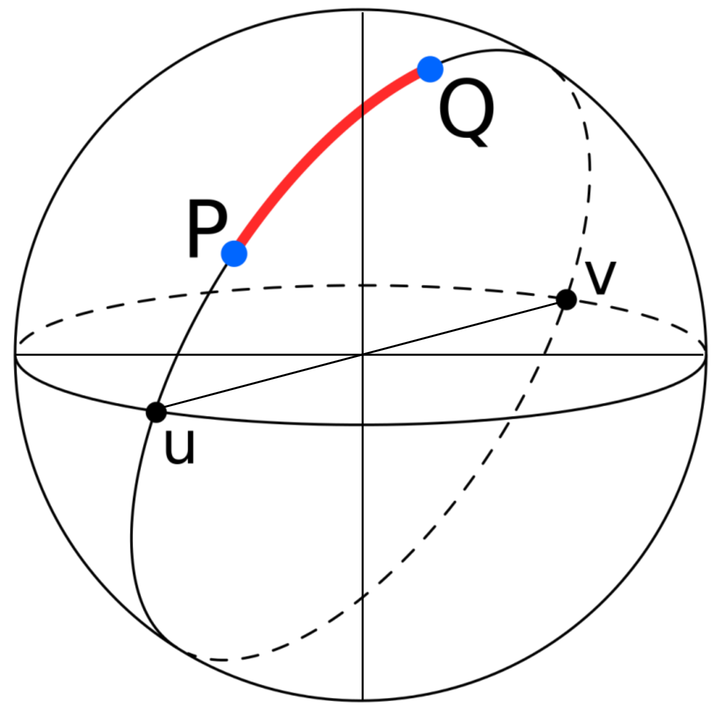
# Geodesia

**<u>Geodésicas</u>**[7,9,10]**:** Longitud más corta entre dos puntos sobre una superficie.

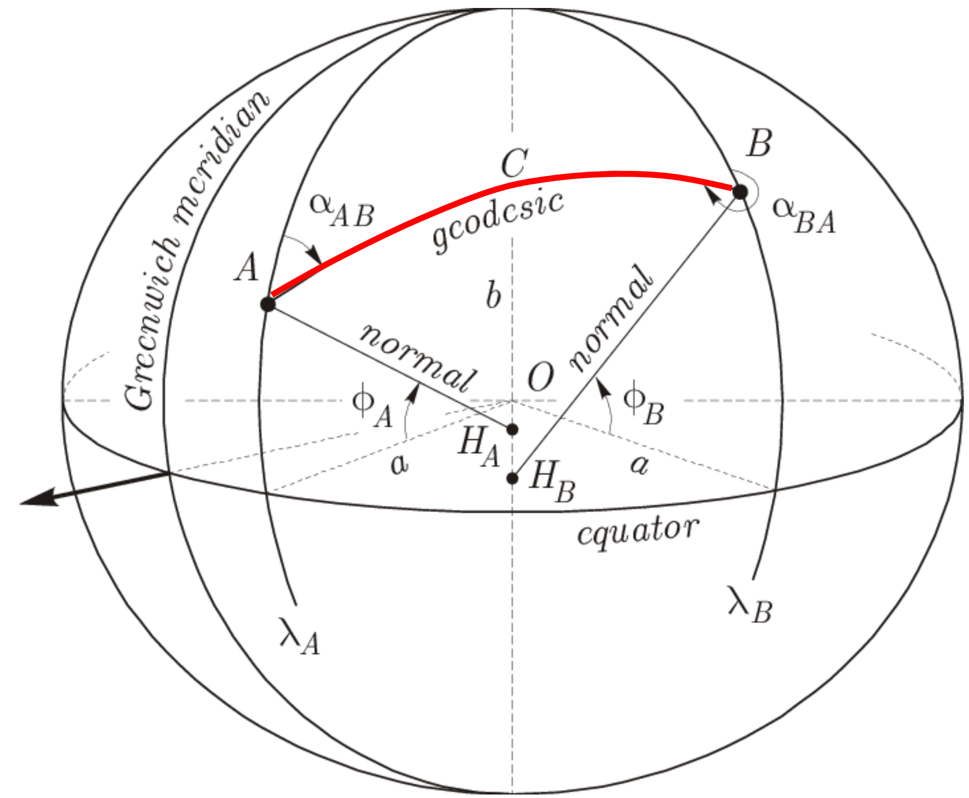
<u>Geodésica en el plano</u>




<u>Geodésica en la esfera</u>




<u>Geodésica en el elipsoide</u>




Fuente: Deakin, R. E., (2005).
Traverse computation: Ellipsoid versus the UTM projection.


**(7) Rapp, Richard H. (1991). *Geometric geodesy part I*. Department of Geodetic Science and Surveying, Ohio State University.**
**(9) Jekeli, C. (2006). *Geometric reference systems in geodesy*. Division of Geodetic Science, School of Earth Sciences. Ohio State University.**
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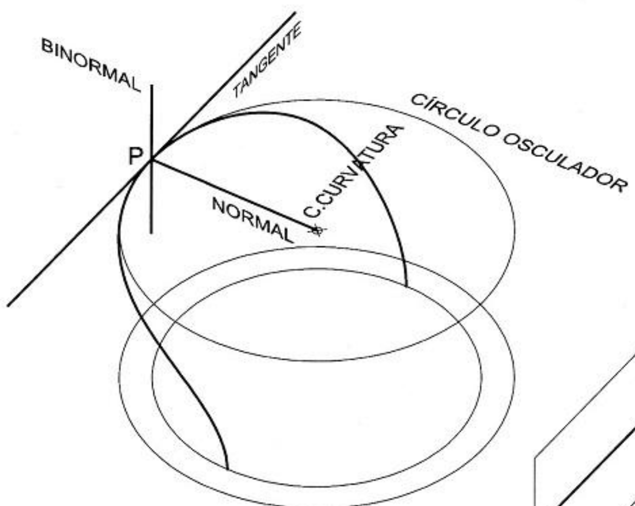
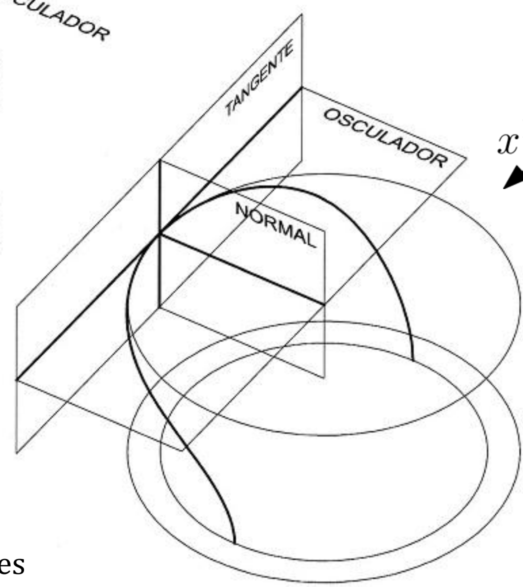
# Geodesia

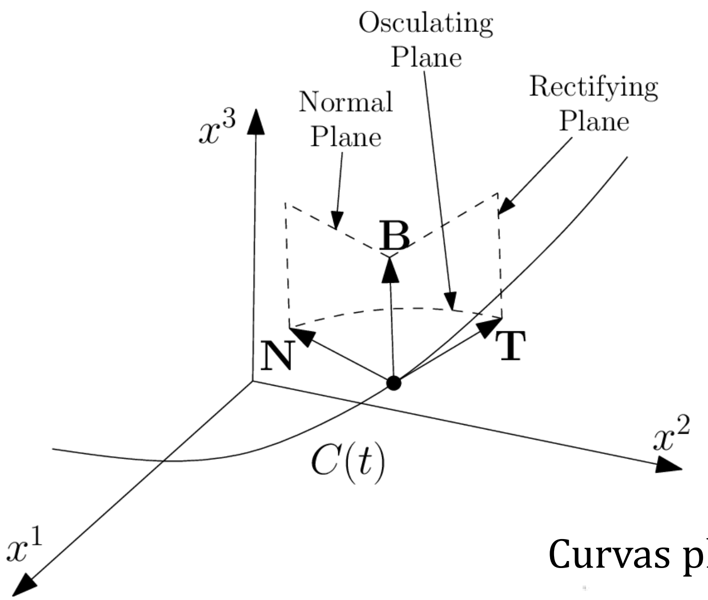
**Geodésicas[7,9,10]:** toda curva sobre una superficie cuyo plano osculador contiene a la normal a la superficie en cualquier punto.

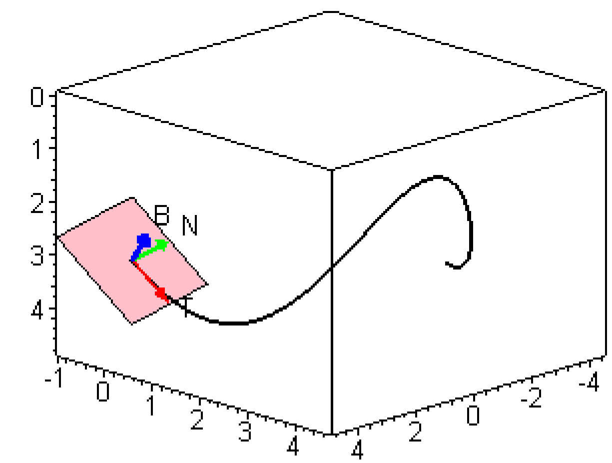
Triedo de Frenet



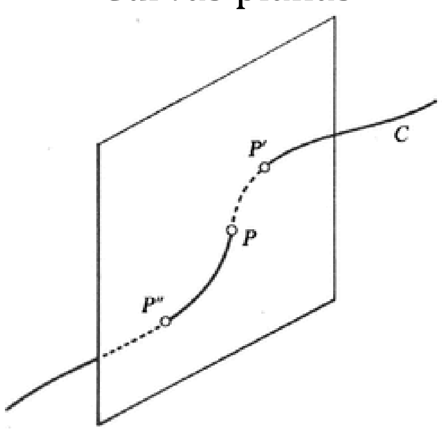

Fuente: http://www.edificacion.upm.es



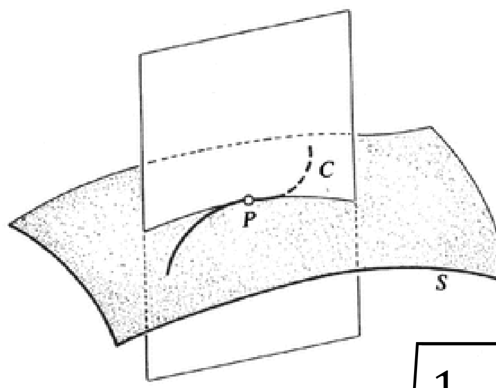




Fuente: http://faculty.etsu.edu

Curvas planas




Doble curvatura

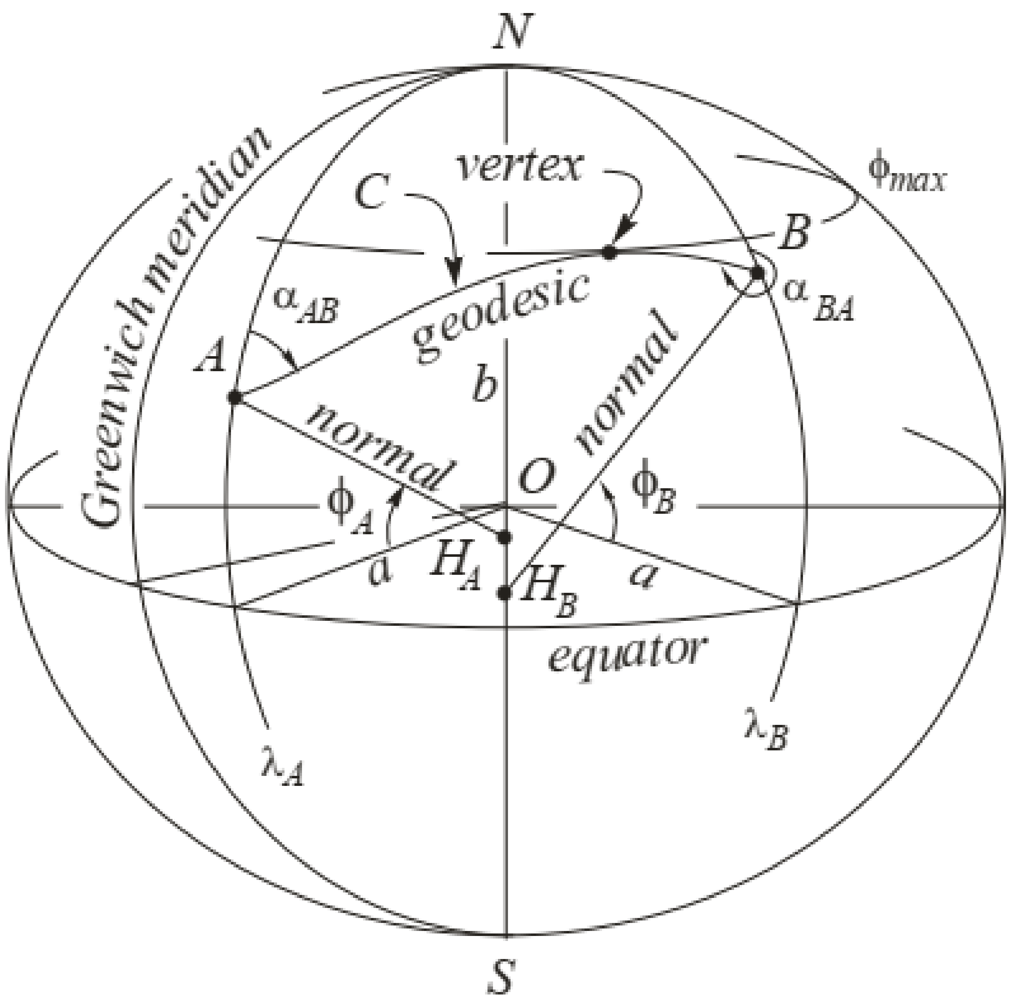



$$\sqrt{\frac{1}{\chi^2}+\frac{1}{\tau^2}}$$

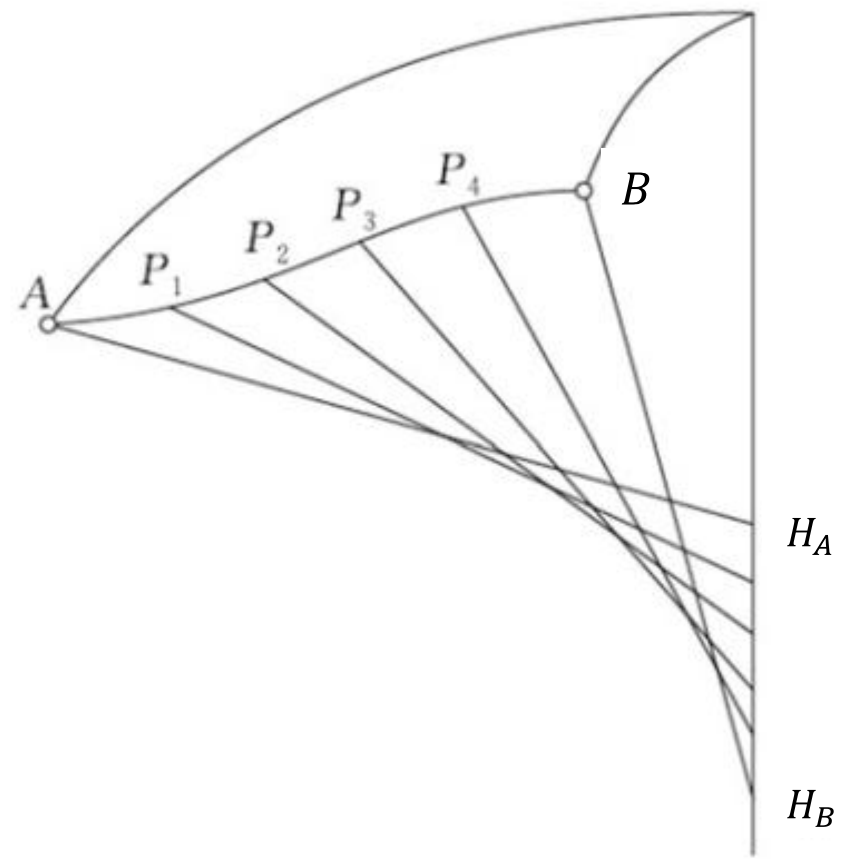
(7) Rapp, Richard H. (1991). *Geometric geodesy part I*. Department of Geodetic Science and Surveying, Ohio State University.
(9) Jekeli, C. (2006). *Geometric reference systems in geodesy*. Division of Geodetic Science, School of Earth Sciences. Ohio State University.
(10) Jordan, W. (1962). *Handbook of Geodesy* (Vol. 3). Corps of Engineers, United States Army, Army Map Service

# Geodesia

**Geodésicas[7,9,10]:** toda curva sobre una superficie cuyo plano osculador contiene a la normal a la superficie en cualquier punto.




Fuente: Deakin, R. E., & Hunter, M. N. (2009).

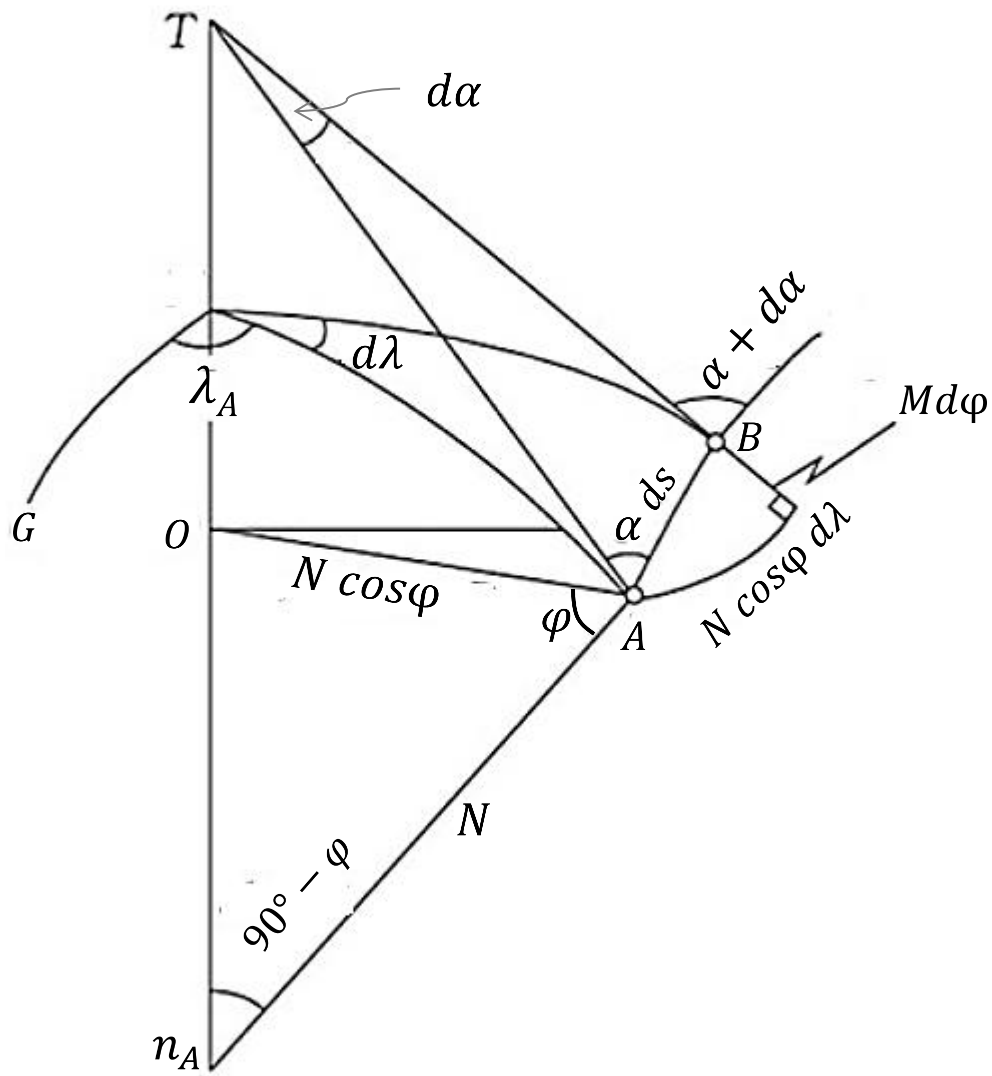



Fuente: Lu Z., Qu Y., Qiao S. (2014)

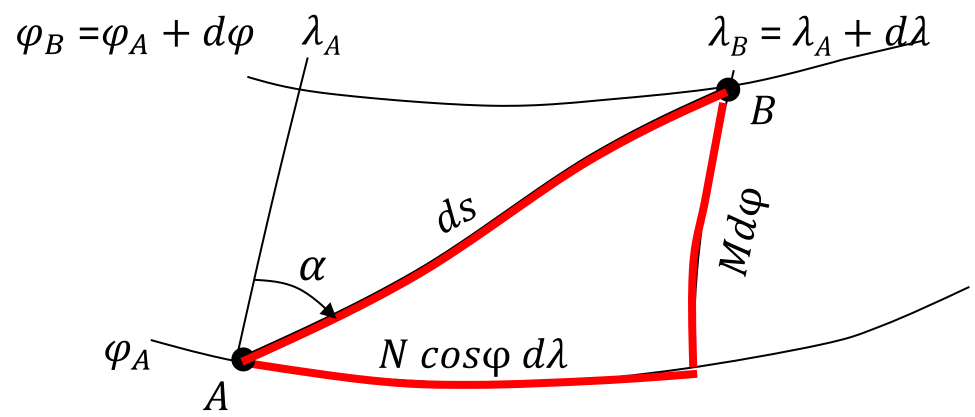

**(7) Rapp, Richard H. (1991).** ***Geometric geodesy part I*****. Department of Geodetic Science and Surveying, Ohio State University.**
**(9) Jekeli, C. (2006).** ***Geometric reference systems in geodesy*****. Division of Geodetic Science, School of Earth Sciences. Ohio State University.**
**(10) Jordan, W. (1962).** ***Handbook of Geodesy*** **(Vol. 3). Corps of Engineers, United States Army, Army Map Service**

# Geodesia

## Ecuación Fundamental de la Geodésica(7,9,10):




Fuente: Lu Z., Qu Y., Qiao S. (2014)




$$\cos\alpha = \frac{Md\varphi}{dS} \rightarrow dS\cos\alpha = Md\varphi$$

$$sen\,\alpha = \frac{N\,cos\varphi\,d\lambda}{dS} \rightarrow dS\,sen\,\alpha = N\,cos\varphi\,d\lambda$$

$$d\alpha = \frac{N\,cos\varphi\,d\lambda}{AT}$$

$$\cos(90° - \varphi) = \frac{N\,cos\varphi}{AT}$$

$$d\alpha = sen\,\varphi\,d\lambda \rightarrow d\alpha = \frac{sen\,\alpha}{N} tg\varphi\,dS$$

Ecuación de Laplace

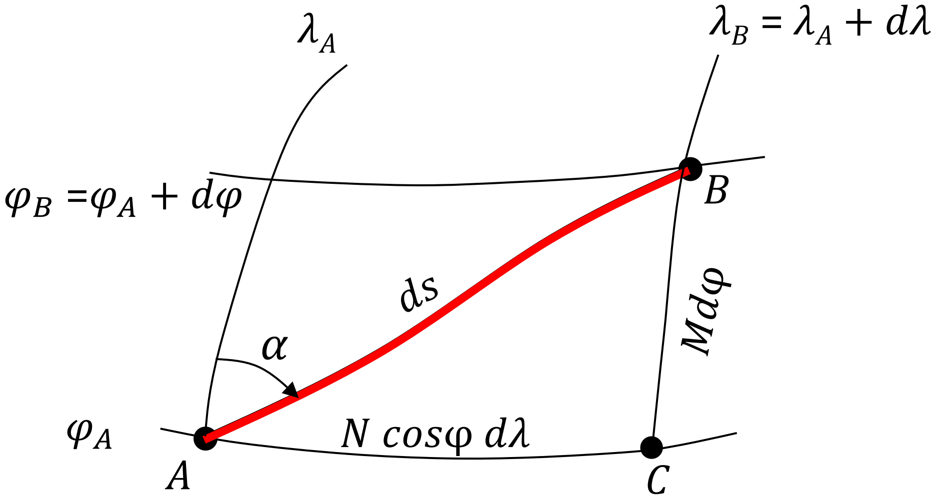
$$dS = \sqrt{(Md\varphi)^2 + (N\,cos\varphi\,d\lambda)^2} \rightarrow I = \int_A^B dS \rightarrow mínima$$

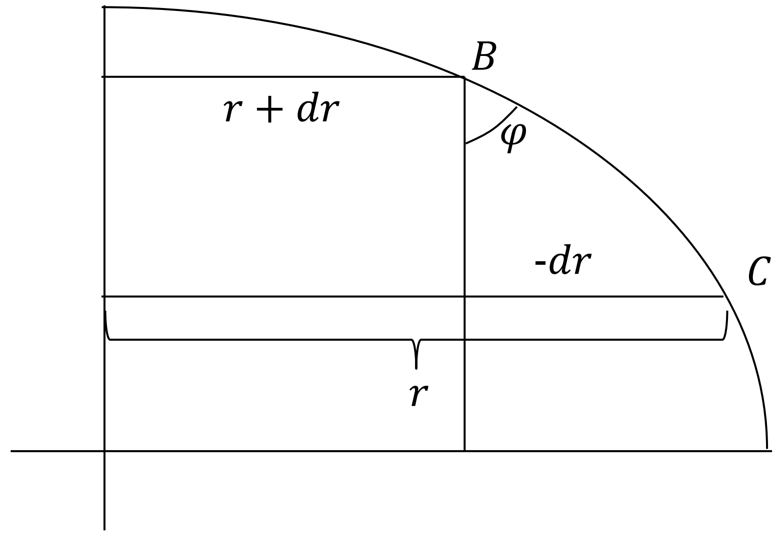

**(7) Rapp, Richard H. (1991). *Geometric geodesy part I*. Department of Geodetic Science and Surveying, Ohio State University.**
**(9) Jekeli, C. (2006). *Geometric reference systems in geodesy*. Division of Geodetic Science, School of Earth Sciences. Ohio State University.**
**(10) Jordan, W. (1962). *Handbook of Geodesy* (Vol. 3). Corps of Engineers, United States Army, Army Map Service**

# Geodesia

## Ecuación Fundamental de la Geodésicas(7,9,10):

Longitud más corta entre dos puntos sobre una superficie. $\rightarrow I = \int_A^B dS \rightarrow mínima$




$$-dr = M \, sen\varphi \, d\varphi$$

$$dS \cos\alpha = M d\varphi$$

$$dS \, sen\,\alpha = N \cos\varphi \, d\lambda = r \, d\lambda$$

$$\cos\alpha = \frac{M d\varphi}{dS} \rightarrow r \, d\alpha \cos\alpha = \frac{M d\varphi}{dS} r \, d\alpha = \frac{M d\varphi}{dS} r \, sen\varphi \, d\lambda$$

$$sen\,\alpha = \frac{N \cos\varphi \, d\lambda}{dS} \rightarrow dr \, sen\,\alpha = r \frac{d\lambda}{dS} dr = -r \frac{d\lambda}{dS} M \, sen\varphi \, d\varphi$$

$$r \, d\alpha \cos\alpha + dr \, sen\,\alpha = 0$$

$$r \, sen\,\alpha = cte$$

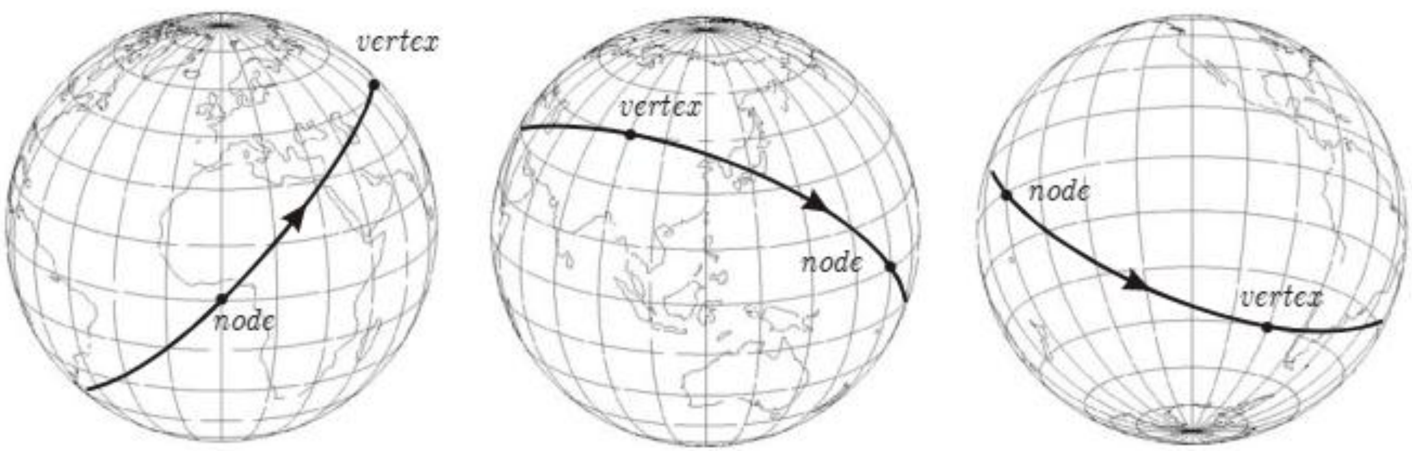
$$N \cos\varphi \, sen\,\alpha = cte$$

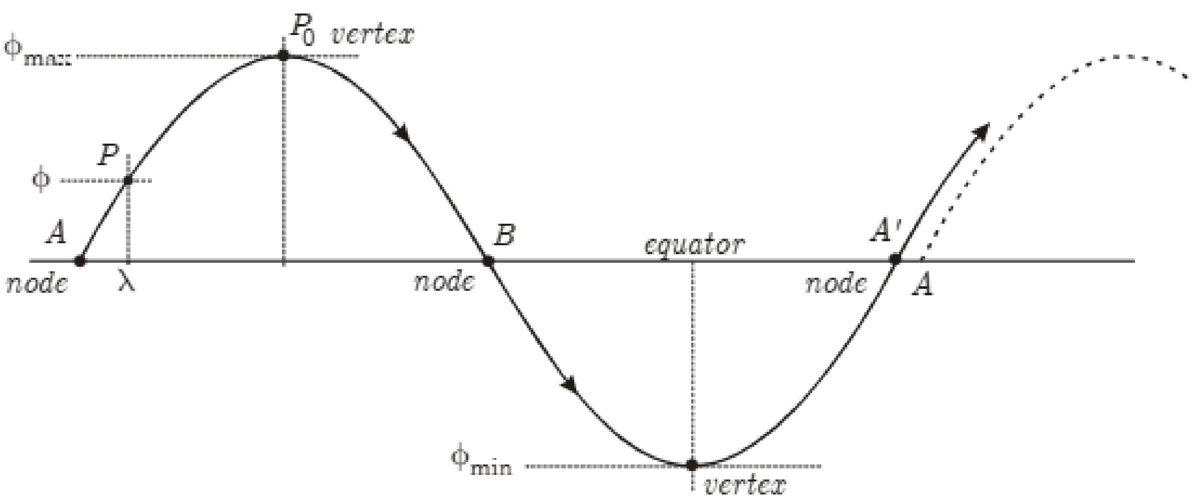
Ecuación de Clairaut


**(7) Rapp, Richard H. (1991).** ***Geometric geodesy part I*****. Department of Geodetic Science and Surveying, Ohio State University.**
**(9) Jekeli, C. (2006).** ***Geometric reference systems in geodesy*****. Division of Geodetic Science, School of Earth Sciences. Ohio State University.**
**(10) Jordan, W. (1962).** ***Handbook of Geodesy*** **(Vol. 3). Corps of Engineers, United States Army, Army Map Service**

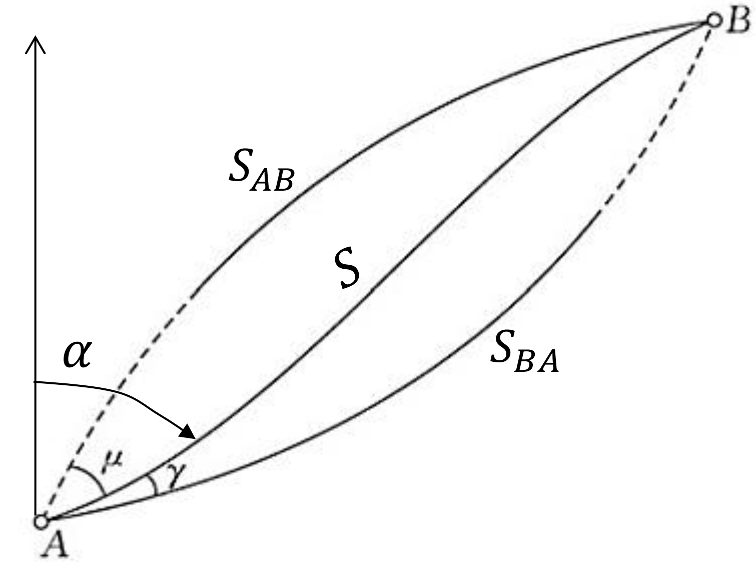
**Geodésicas[7,9,10]**: Longitud más corta entre dos puntos sobre una superficie.

Ecuación de Clairaut $N\cos\varphi \, sen\,\alpha = cte$




Fuente: Deakin, R. E., & Hunter, M. N. (2009).




Fuente: Lu Z., Qu Y., Qiao S. (2014)

$$N\cos\varphi = a\cos\beta$$

$$a\cos\beta \, sen\,\alpha = k$$

Sí $\beta$=0° → $sen\,\alpha_E = \frac{k}{a}$

(7) Rapp, Richard H. (1991). *Geometric geodesy part I*. Department of Geodetic Science and Surveying, Ohio State University.
(9) Jekeli, C. (2006). *Geometric reference systems in geodesy*. Division of Geodetic Science, School of Earth Sciences. Ohio State University.
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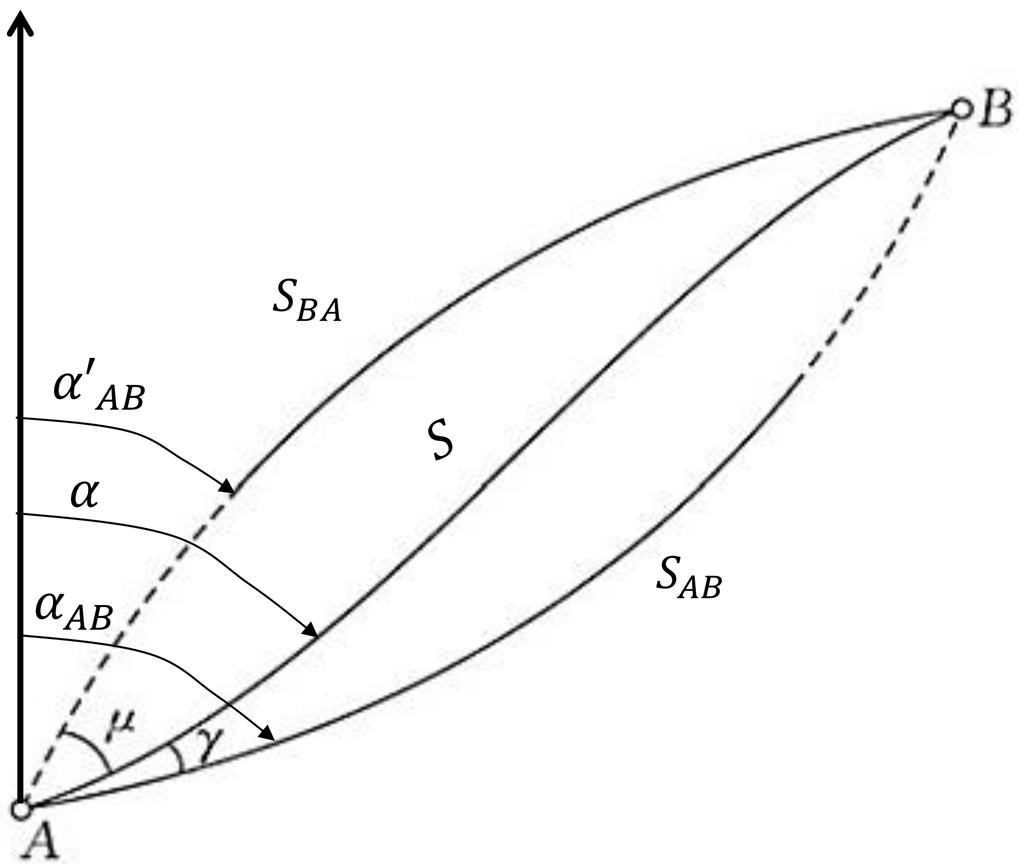
## Diferencia entre sección normal y Geodésica[7,9,10]:




Fuente: Lu Z., Qu Y., Qiao S. (2014)

Diferencia de azimut entre secciones recíprocas

$$\Delta = \alpha_{AB} - \alpha'_{AB} = \frac{e^2}{4}\left(\frac{S_{AB}}{N_A}\right)^2 sen2\alpha_{AB}\, cos^2\varphi_m$$

Diferencia de azimut entre sección normal y geodésica

$$\gamma = \alpha_{AB} - \alpha = \frac{\eta^2}{2}\left(\frac{S_{AB}}{N_A}\right)^2 sen\alpha_{AB}\, cos\alpha_{AB} + \frac{\eta^2 t}{24}\left(\frac{S_{AB}}{N_A}\right)^3 sen\alpha_{AB} \cong \frac{\Delta}{3}$$

Diferencia de longitud entre sección normal y geodésica

$$S_{AB} - S = \frac{a\, e^4}{360}\left(\frac{S_{AB}}{N_A}\right)^5 sen^2 2\alpha\, cos^4\varphi$$

**(7) Rapp, Richard H. (1991). *Geometric geodesy part I*. Department of Geodetic Science and Surveying, Ohio State University.**
**(9) Jekeli, C. (2006). *Geometric reference systems in geodesy*. Division of Geodetic Science, School of Earth Sciences. Ohio State University.**
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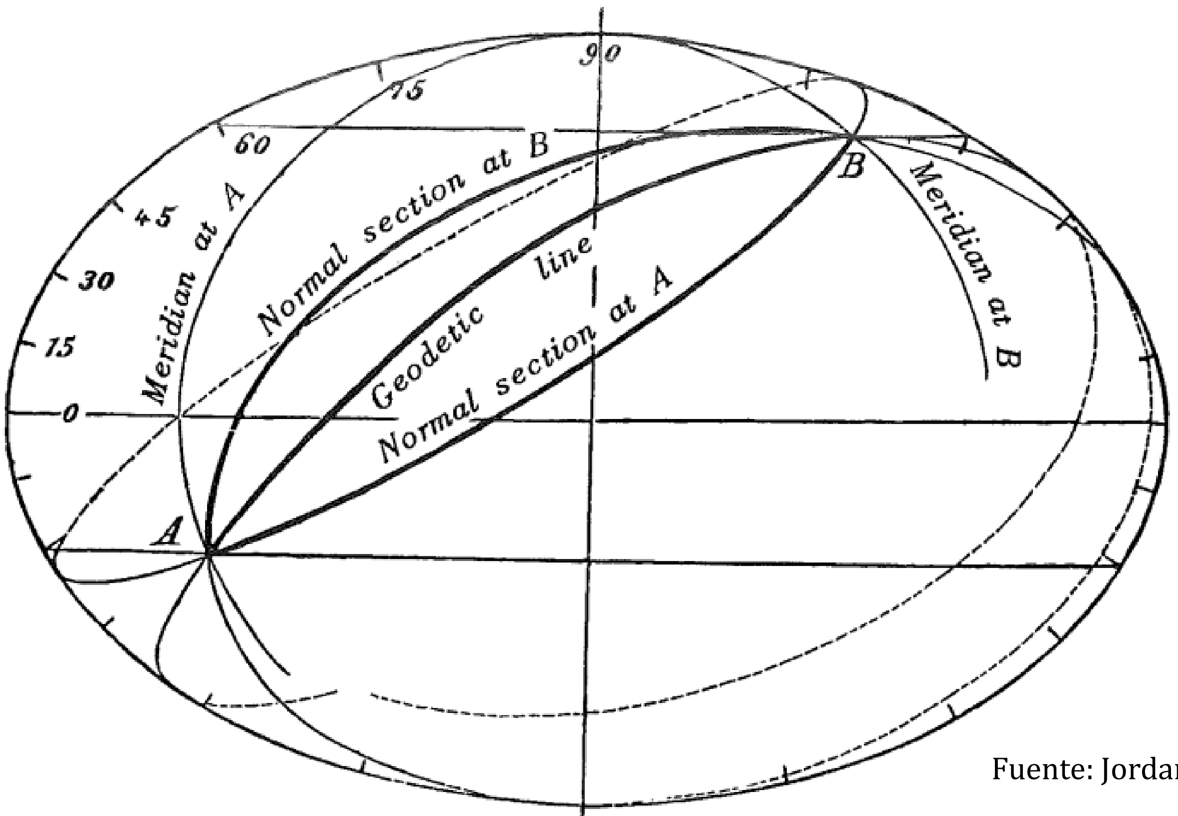
# Geodesia

## Consecuencias[7,9,10]:

- La normal principal de la geodésica es normal al elipsoide en cada punto de la geodésica;
- Cualquier meridiano es una geodésica;
- El ecuador es una geodésica, excepto para puntos diametralmente opuestos;
- Los círculos paralelos no son geodésicas;
- Una geodésica en el elipsoide es una curva de doble curvatura, excepto dos casos particulares;
- Una geodésica alcanza latitudes máximas y mínimas ($\varphi_{máx} = -\varphi_{mín}$);




Fuente: Jordan, W. (1962).

(7) Rapp, Richard H. (1991). *Geometric geodesy part I*. Department of Geodetic Science and Surveying, Ohio State University.
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