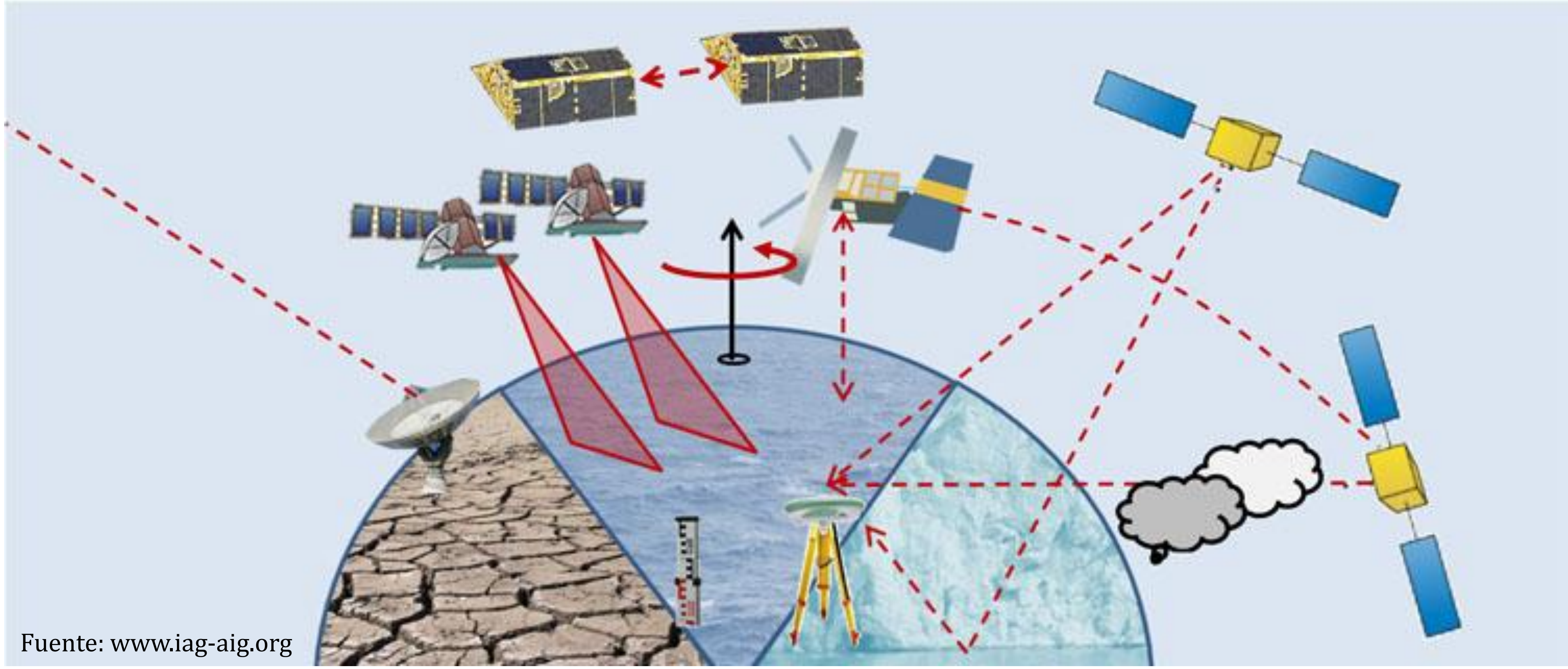
GEODESIA, PERO… ¿PARA QUÉ?



Fuente: www.iag-aig.org

Transporte de coordenadas[11]:

Cálculo de elementos de un triángulo

Cantidades observadas → $a, \beta, \gamma, \overline{AB}$

$$\frac{sen\alpha}{sen\gamma} = \frac{a}{\overline{AB}}$$

$$b^2 = a^2 + \overline{AB}^2 - 2a\overline{AB}\cos\beta$$

Obtención de lados y ángulos de *ABP*

Agregamos otras cantidades → $Az, (x, y)_A$

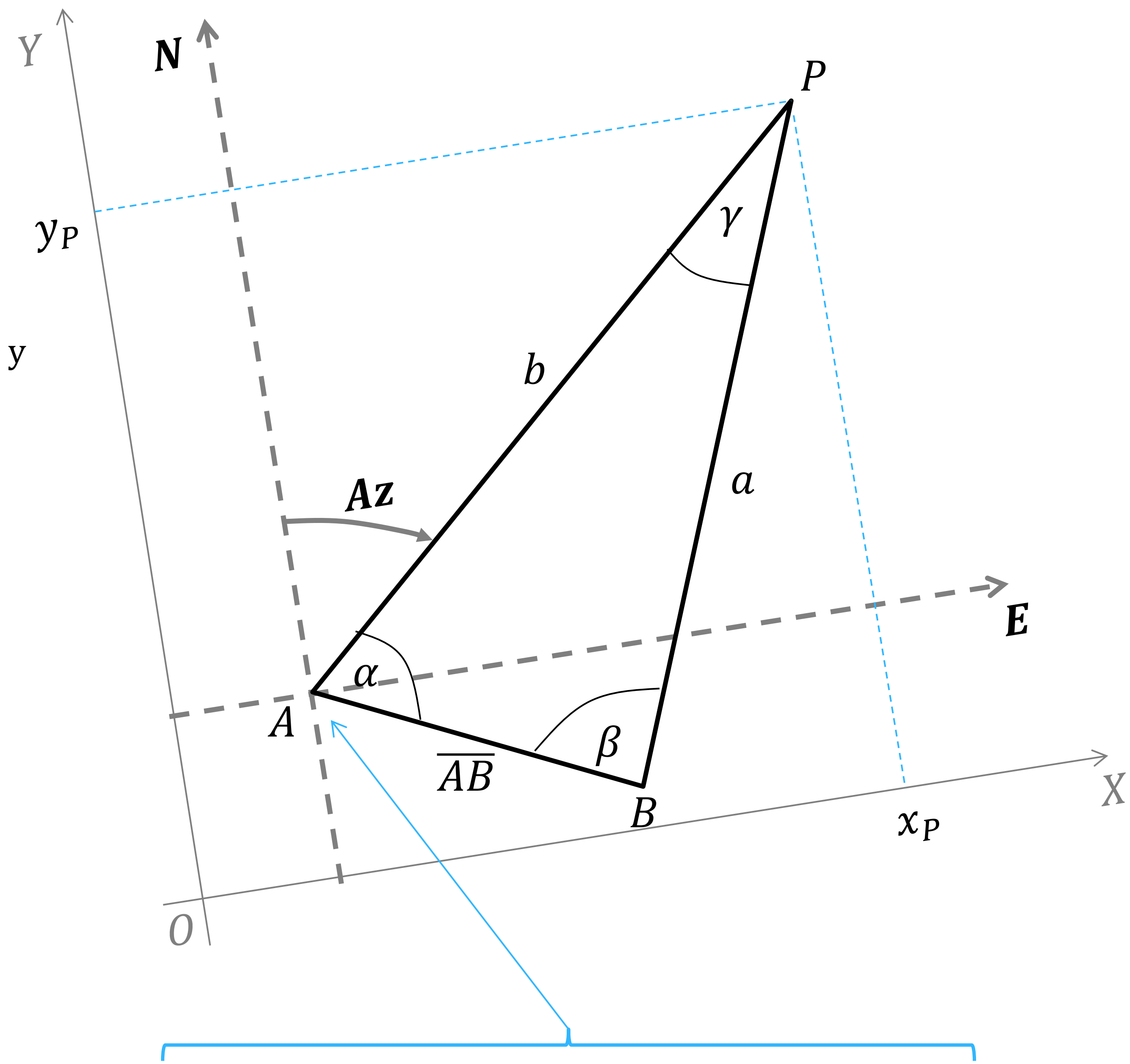
$$x_P = b\ senAz + x_A$$

$$y_P = b\ cosAz + y_A$$

Cantidades observadas → $(x, y)_A; (x, y)_P$

$$tgAz = \frac{x_P - x_A}{y_P - y_A}$$

$$b = \frac{x_P - x_A}{senAz} = \frac{y_P - y_A}{cosAz}$$

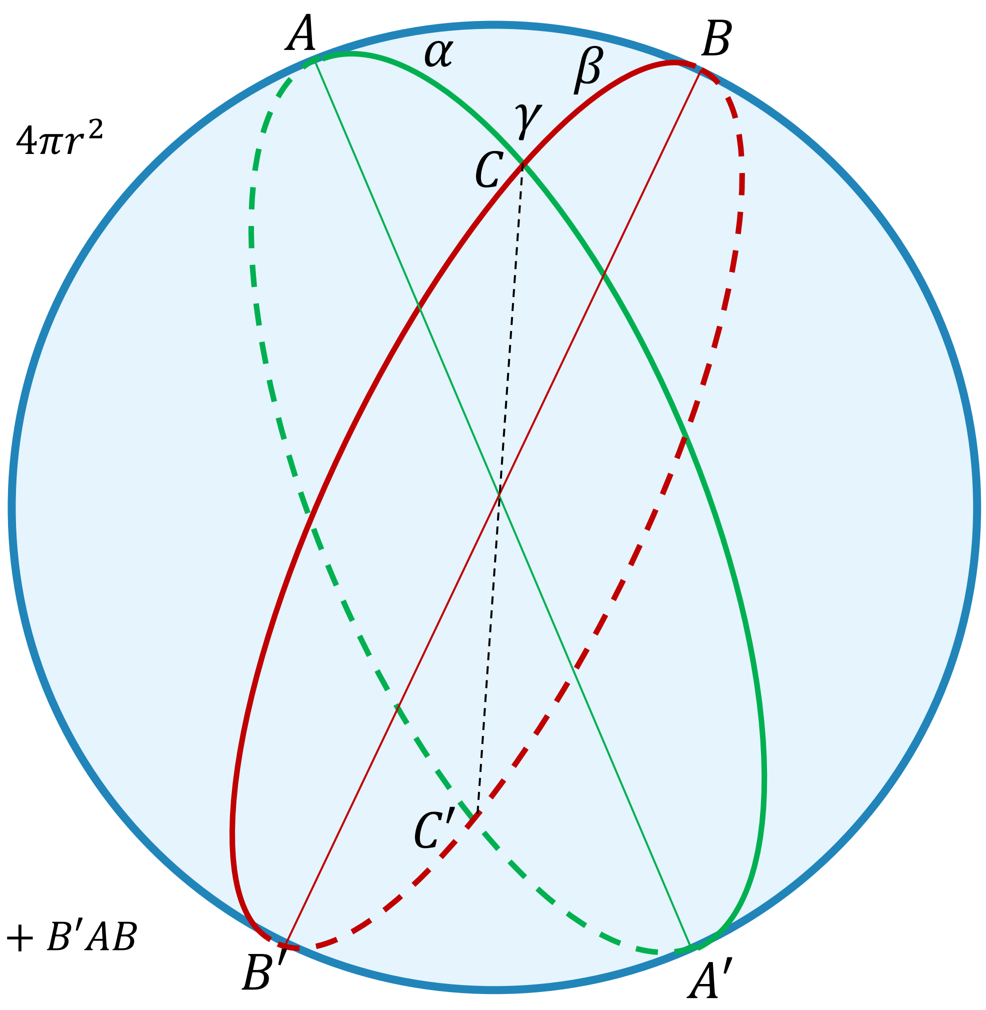


Punto Fundamental Astronómico → Φ, Λ, Az

(11) Domínguez García-Tejero, F. (1998). *Topografía general y aplicada*.

Geodesia

Solución de triángulos elipsoidales[7,9,10]:



$Sup = 4\pi r^2$

Exceso esférico → $\varepsilon = (\alpha + \beta + \gamma) - 180°$

Teorema fundamental → $\varepsilon = \frac{ST}{R^2}$

$$Sup\ AA' = \frac{\alpha}{360}(4\pi r^2)$$
$$Sup\ BB' = \frac{\beta}{360}(4\pi r^2)$$
$$Sup\ CC' = \frac{\gamma}{360}(4\pi r^2)$$

$$Sup\ AA' + Sup\ BB' + Sup\ CC' = \frac{\alpha + \beta + \gamma}{360}(4\pi r^2)$$

$$Sup\ AA' = ST + A'BC$$
$$Sup\ BB' = ST + B'AC$$
$$Sup\ CC' = ST + B'AB$$

$$Sup\ AA' + Sup\ BB' + Sup\ CC' = 3\ ST + A'BC + B'AC + B'AB$$

$$3\ ST = 2\ ST + ST$$
$$ST + A'BC + B'AC + B'AB = 2\pi r^2$$

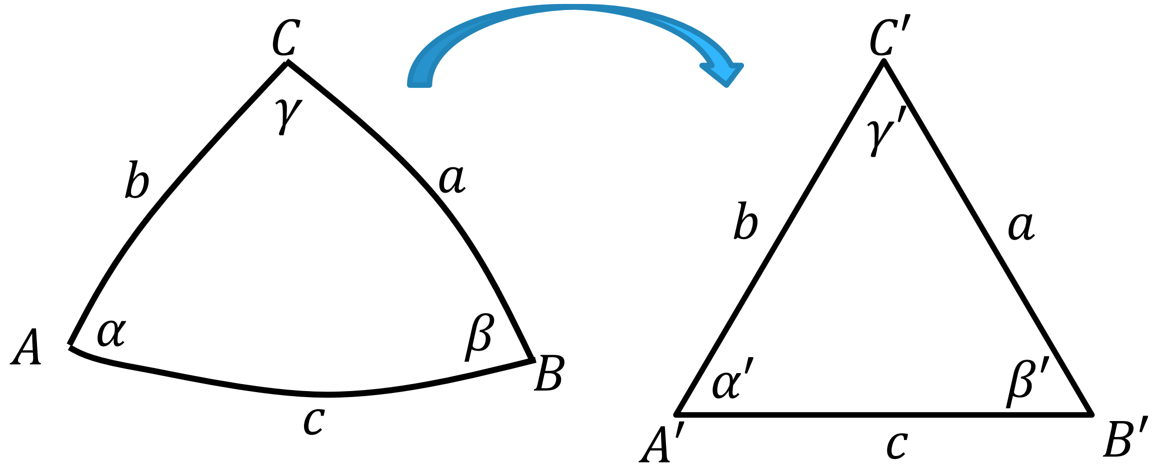
$$Sup\ AA' + Sup\ BB' + Sup\ CC' = 2\ ST + 2\pi r^2$$

$$\frac{\alpha + \beta + \gamma}{360}(4\pi r^2) = 2\ ST + 2\pi r^2 \longrightarrow ST = (\alpha + \beta + \gamma - 180°)\frac{\pi}{180°}r^2$$

(7) Rapp, Richard H. (1991). *Geometric geodesy part I*. Department of Geodetic Science and Surveying, Ohio State University.
(9) Jekeli, C. (2006). *Geometric reference systems in geodesy*. Division of Geodetic Science, School of Earth Sciences. Ohio State University.
(10) Jordan, W. (1962). *Handbook of Geodesy* (Vol. 3). Corps of Engineers, United States Army, Army Map Service

Geodesia

Solución de triángulos elipsoidales[7,9,10]:



Teorema de Legendre: se asumen lados del triángulo esférico y del triángulo auxiliar iguales, entonces los ángulos del triángulo auxiliar varían en un tercio del exceso esférico.

$$\cos\frac{a}{R} = \cos\frac{b}{R}\cos\frac{c}{R} + sen\frac{b}{R}\,sen\frac{c}{R}\cos\alpha$$

$$a^2 = b^2 + c^2 - 2bc\cos\alpha'$$

$$\alpha - \alpha' = \frac{p}{3R^2} \qquad \beta - \beta' = \frac{p}{3R^2} \qquad \gamma - \gamma' = \frac{p}{3R^2}$$

$$\rightarrow \alpha + \beta + \gamma = 180° + \frac{p}{R^2}$$

$$\alpha - \alpha' = \frac{\varepsilon}{3} \qquad \beta - \beta' = \frac{\varepsilon}{3} \qquad \gamma - \gamma' = \frac{\varepsilon}{3}$$

Aproximación esférica

$$\varepsilon = \frac{p}{R^2}\left(1 + \frac{a^2 + b^2 + c^2}{24R^2}\right)$$

$$p = \sqrt{s(s-a)(s-b)(s-c)}$$

$$s = \frac{a+b+c}{2}$$

$$m^2 = \frac{a^2 + b^2 + c^2}{3}$$

$$\alpha - \alpha' = \frac{\varepsilon}{3} + \frac{\varepsilon}{60R^2}(m^2 - a^2)$$

$$\beta - \beta' = \frac{\varepsilon}{3} + \frac{\varepsilon}{60R^2}(m^2 - b^2)$$

$$\gamma - \gamma' = \frac{\varepsilon}{3} + \frac{\varepsilon}{60R^2}(m^2 - c^2)$$

Aproximación elipsóidica

$$K_A = (MN)_A^{\ -1};\ K_B = (MN)_B^{\ -1};\ K_C = (MN)_C^{\ -1}$$

$$K_m = (K_A + K_B + K_A)/3 \qquad \varepsilon = pK_m\left(1 + \frac{m^2 K_m}{8}\right)$$

$$\alpha - \alpha' = \frac{\varepsilon}{3} + \frac{\varepsilon}{60}K_m(m^2 - a^2) + \frac{\varepsilon}{12}\frac{K_A - K_m}{K_m}$$

$$\beta - \beta' = \frac{\varepsilon}{3} + \frac{\varepsilon}{60}K_m(m^2 - b^2) + \frac{\varepsilon}{12}\frac{K_B - K_m}{K_m}$$

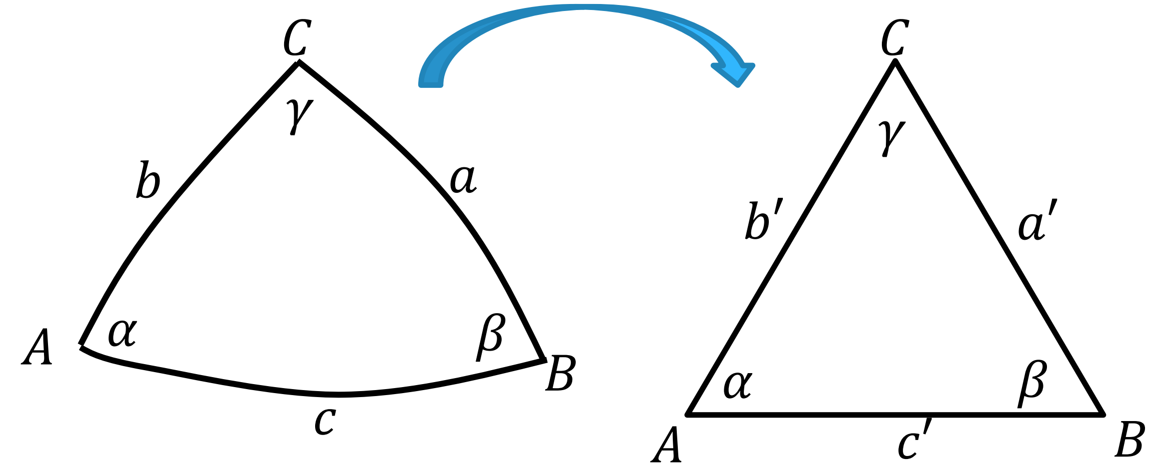
$$\gamma - \gamma' = \frac{\varepsilon}{3} + \frac{\varepsilon}{60}K_m(m^2 - c^2) + \frac{\varepsilon}{12}\frac{K_C - K_m}{K_m}$$

(7) Rapp, Richard H. (1991). ***Geometric geodesy part I*****. Department of Geodetic Science and Surveying, Ohio State University.**
(9) Jekeli, C. (2006). ***Geometric reference systems in geodesy*****. Division of Geodetic Science, School of Earth Sciences. Ohio State University.**
(10) Jordan, W. (1962). ***Handbook of Geodesy*** **(Vol. 3). Corps of Engineers, United States Army, Army Map Service**

Geodesia

Solución de triángulos elipsoidales(7,9,10)**:**

Cantidades adicionales: cantidad que modifica los lados del triángulo esférico.



$$\frac{sen\ \alpha}{sen\ \beta} = \frac{sen\frac{a}{R}}{sen\frac{b}{R}}$$

$$\frac{sen\ \alpha}{sen\ \beta} = \frac{a'}{b'}$$

$$\frac{a'}{b'} = \frac{sen\frac{a}{R}}{sen\frac{b}{R}} = \frac{a - \frac{a^3}{6R^2}}{b - \frac{b^3}{6R^2}} \rightarrow \begin{cases} a' = a - \frac{a^3}{6R^2} \\ b' = b - \frac{b^3}{6R^2} \end{cases} \rightarrow l' = l - \frac{l^3}{6R^2}$$

aditamentos

$$b' = a'\frac{sen\ \beta}{sen\ \alpha}$$

$$b = b' + \frac{b^3}{6R^2}$$

$$b = a'\frac{sen\ \beta}{sen\ \alpha} + \frac{b^3}{6R^2} \approx b' + \frac{b'^3}{6R^2}$$

$$c' = a'\frac{sen\ \gamma}{sen\ \alpha} \rightarrow c = a'\frac{sen\ \gamma}{sen\ \alpha} + \frac{c^3}{6R^2} \approx c' + \frac{c'^3}{6R^2}$$

(7) Rapp, Richard H. (1991). *Geometric geodesy part I*. Department of Geodetic Science and Surveying, Ohio State University.
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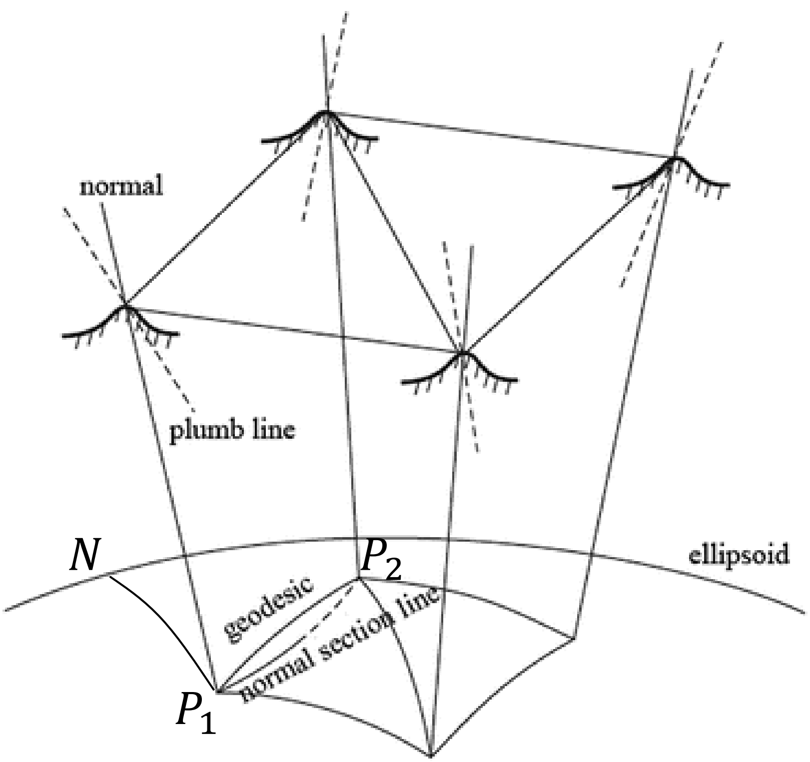
Geodesia

Problema Fundamental de la Geodesia(7,9,10):

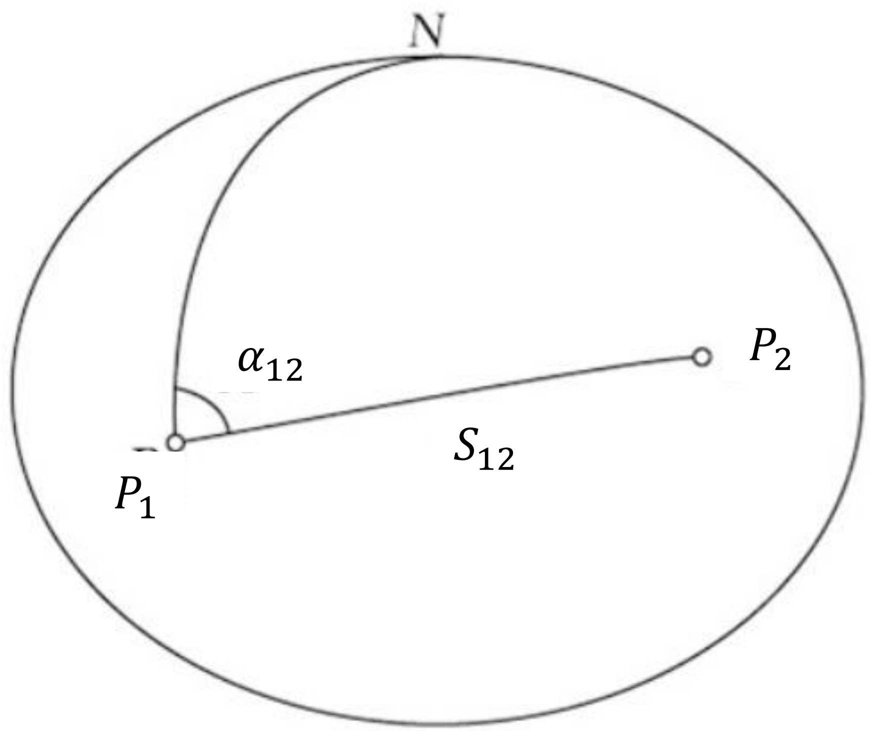
Cálculo de coordenadas, direcciones y distancias sobre el elipsoide

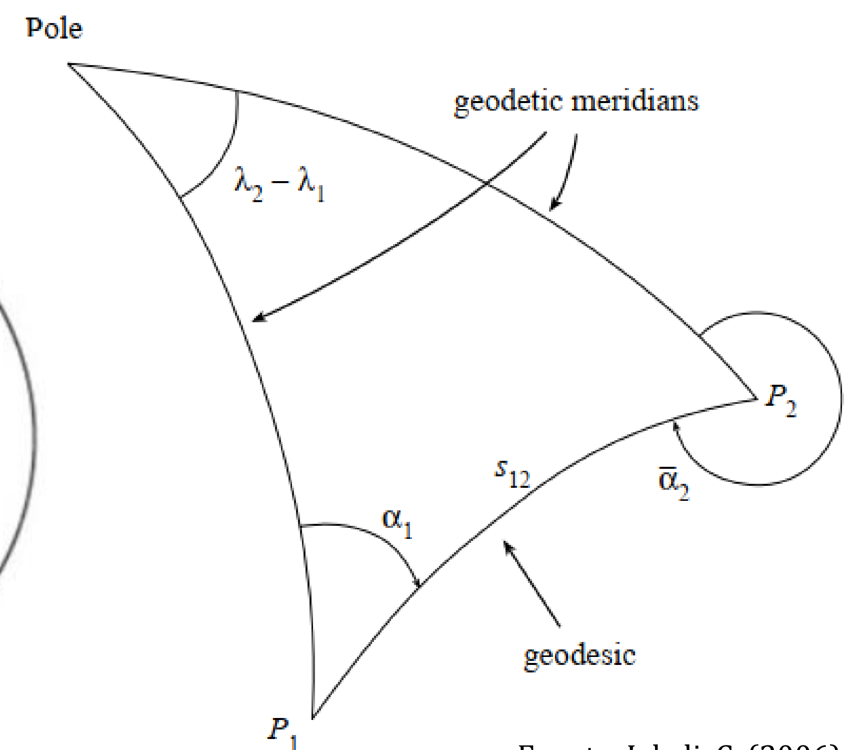
PFG
- Problema Geodésico Directo (PGD) → $\varphi_1, \lambda_1, \alpha_1, S_{12} \rightarrow \varphi_2, \lambda_2, \bar{\alpha}_2$
- Problema Geodésico Inverso (PGI) → $\varphi_1, \lambda_1, \varphi_2, \lambda_2 \rightarrow \alpha_1, \bar{\alpha}_2, S_{12}$

Líneas cortas ($S < 200\ Km$)
Líneas medias (200 Km $< S < 1000\ Km$)
Líneas largas (1000 Km $< S < 2000\ Km$)



Fuente: Lu Z., Qu Y., Qiao S. (2014)





Fuente: Jekeli, C. (2006)

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(9) Jekeli, C. (2006). *Geometric reference systems in geodesy*. Division of Geodetic Science, School of Earth Sciences. Ohio State University.
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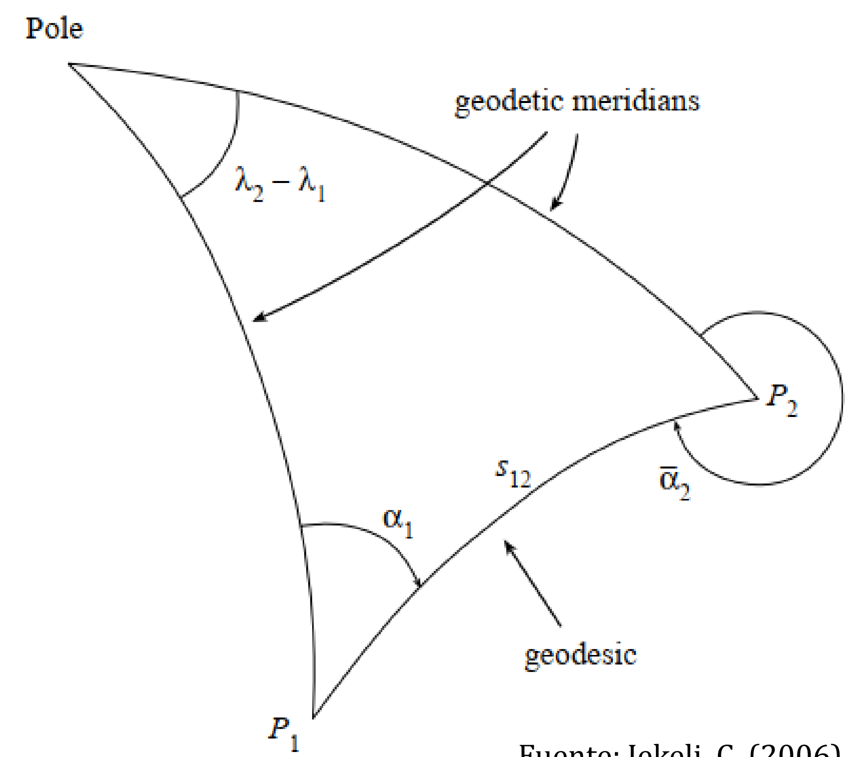
Líneas cortas ($S < 200\ Km$)
Líneas medias ($200\ \text{Km} < S < 1000\ Km$)
Líneas largas ($1000\ \text{Km} < S < 2000\ Km$)

Soluciones
- Desarrollos en serie e iteraciones
- Esfera auxiliar
- Integración numérica



Fuente: Jekeli, C. (2006)

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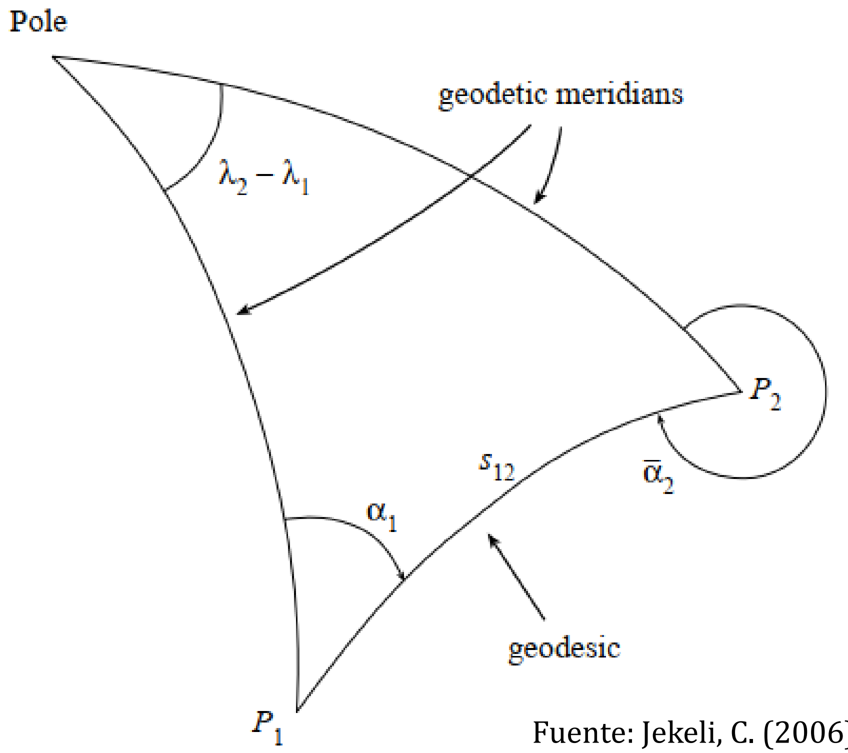
Geodesia

Problema Fundamental de la Geodesia(7,9,10):

Problema Geodésico Directo (PGD) → $\varphi_1, \lambda_1, \alpha_1, S_{12} \rightarrow \varphi_2, \lambda_2, \bar{\alpha}_2$



Fuente: Jekeli, C. (2006)

$$\varphi = \varphi(S), \qquad \lambda = \lambda(S), \qquad \alpha = \alpha(S)$$

$$\left\{\begin{aligned} \varphi_2 - \varphi_1 &= \left.\frac{d\varphi}{dS}\right|_1 S_{12} + \frac{1}{2!}\left.\frac{d^2\varphi}{dS^2}\right|_1 S^2{}_{12} + \ \dots \\ \lambda_2 - \lambda_1 &= \left.\frac{d\lambda}{dS}\right|_1 S_{12} + \frac{1}{2!}\left.\frac{d^2\lambda}{dS^2}\right|_1 S^2{}_{12} + \ \dots \\ (\bar{\alpha}_2 - \alpha_1) - \pi &= \left.\frac{d\alpha}{dS}\right|_1 S_{12} + \frac{1}{2!}\left.\frac{d^2\alpha}{dS^2}\right|_1 S^2{}_{12} + \ \dots \end{aligned}\right.$$

$$\left\{\begin{aligned} \cos\alpha &= \frac{M d\varphi}{dS} & &\rightarrow \frac{d\varphi}{dS} = \frac{\cos\alpha}{M} \\ sen\,\alpha &= \frac{N\,cos\varphi\,d\lambda}{dS} & &\rightarrow \frac{d\lambda}{dS} = \frac{sen\,\alpha}{N\,cos\varphi} \\ d\alpha &= sen\,\varphi\,d\lambda & &\rightarrow \frac{d\alpha}{dS} = \frac{sen\,\alpha}{N\,cos\varphi} \end{aligned}\right.$$

Algunas variables útiles:

$$V^2 = 1 + \eta^2$$

$$\eta^2 = e'^2 cos^2\varphi_1$$

$$t = tg\varphi_1$$

$$u = \frac{S_{12}\,sen\alpha_1}{N_1}$$

$$v = \frac{S_{12} cos\alpha_1}{N_1}$$

$$\rho^\circ = \frac{180^\circ}{\pi}$$

$$\rho'' = \frac{180^\circ}{\pi} 3600$$

(7) Rapp, Richard H. (1991). *Geometric geodesy part I*. Department of Geodetic Science and Surveying, Ohio State University.
(9) Jekeli, C. (2006). *Geometric reference systems in geodesy*. Division of Geodetic Science, School of Earth Sciences. Ohio State University.
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Problema Geodésico Directo[7,9,10]: → $\varphi_1, \lambda_1, \alpha_1, S_{12} \rightarrow \varphi_2, \lambda_2, \bar{\alpha}_2$

$$\frac{\varphi_2-\varphi_1}{V^2} = u - \frac{1}{2\rho}v^2t - \frac{3}{2\rho}u^2v^2t - \frac{v^2u}{6\rho^2}(1+3t^2+\eta^2-9\eta^2t^2) - \frac{u^3}{2\rho^2}\eta^2(1-t^2)$$
$$+\frac{v^4t}{24\rho^3}(1+3t^2+\eta^2-9\eta^2t^2) - \frac{v^2u^2}{12\rho^3}t(4+6t^2-13\eta^2-9\eta^2t^2) + \frac{u^4}{2\rho^3}\eta^2t$$
$$+\frac{v^4u}{120\rho^4}(1+30t^2+45t^4) - \frac{v^2u^3}{30\rho^4}(2+15t^2+15t^4)$$

$$(\lambda_2-\lambda_1)cos\varphi_1 = v + \frac{1}{\rho}vut - \frac{v^3t^2}{3\rho^2} + \frac{u^2v}{3\rho^2}(1+3t^2+\eta^2)$$
$$-\frac{v^3u\,t}{3\rho^3}(1+3t^2+\eta^2) + \frac{u^3v\,t}{3\rho^3}(2+3t^2+\eta^2)$$
$$+\frac{v^5t^2}{15\rho^4}(1+3t^2) + \frac{u^4v}{15\rho^4}(2+15t^2+15t^4) - \frac{v^3u^2}{15\rho^4}(1+20t^2+30t^4)$$

$$(\bar{\alpha}_2-\alpha_1)-\pi = vt + \frac{u\,v}{2\rho}(1+2t^2+\eta^2) - \frac{v^3t}{6\rho^2}(1+2t^2+\eta^2) + \frac{u^2v\,t}{6\rho^2}(5+6t^2+\eta^2-4\eta^2)$$
$$-\frac{v^3u}{24\rho^3}(1+20t^2+24t^4+2\eta^2+8\eta^2t^2) + \frac{u^3v}{24\rho^3}(5+28t^2+24t^4+6\eta^2+8\eta^2t^2)$$
$$+\frac{v^5t}{120\rho^4}(1+20t^2+24t^4) - \frac{v^3u^2}{120\rho^4}(58+280t^2+240t^4) + \frac{vu^4}{120\rho^4}(61+180t^2+120t^4)$$

(7) Rapp, Richard H. (1991). *Geometric geodesy part I*. Department of Geodetic Science and Surveying, Ohio State University.
(9) Jekeli, C. (2006). *Geometric reference systems in geodesy*. Division of Geodetic Science, School of Earth Sciences. Ohio State University.
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Problema Geodésico Inverso[7,9,10]: → $\varphi_1, \lambda_1, \varphi_2, \lambda_2 \rightarrow \alpha_1, \bar{\alpha}_2, S_{12}$

$$\Delta\varphi = \varphi_2 - \varphi_1 = V^2 u + \delta\varphi \longrightarrow u = \frac{\Delta\varphi - \delta\varphi}{V^2}$$

$$\Delta\lambda = \lambda_2 - \lambda_1 = \frac{v}{cos\varphi_1} + \delta\lambda \longrightarrow v = (\Delta\lambda - \delta\lambda)cos\varphi_1$$

$$\alpha_1 = tg^{-1}\left(V^2 \frac{\Delta\lambda - \delta\lambda}{\Delta\varphi - \delta\varphi} cos\varphi_1\right)$$

$$S_{12} = \frac{N_1\, cos\varphi_1}{sen\alpha_1}(\Delta\lambda - \delta\lambda)$$

$$S_{12} = \frac{N_1}{cos\alpha_1}\frac{\Delta\varphi - \delta\varphi}{V^2}$$

$\alpha_1 = ?$

→ $\delta\varphi = 0; \quad \delta\lambda = 0$

$$u^0 = \frac{\Delta\varphi}{V^2}$$

$$v^0 = (\Delta\lambda)cos\varphi_1$$

$$\alpha_1{}^0 = tg^{-1}\left(V^2 \frac{\Delta\lambda}{\Delta\varphi} cos\varphi_1\right)$$

$$S_{12}{}^0 = \frac{N_1\, cos\varphi_1}{sen\alpha_1}(\Delta\lambda)$$

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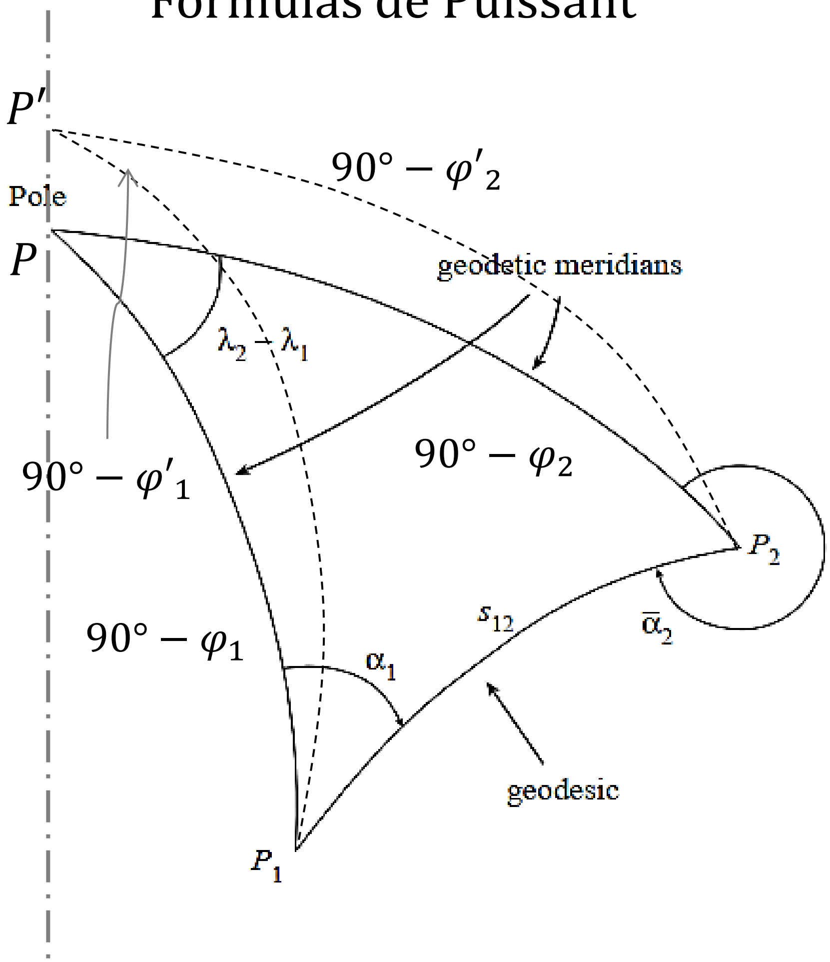
Geodesia

Problema Geodésico Directo(7,9,10): → $\varphi_1, \lambda_1, \alpha_1, S_{12} \rightarrow \varphi_2, \lambda_2, \bar{\alpha}_2$

Fórmulas de Puissant



- Esfera de radio N_1, tangente en P_1
- Casi coincidente en P_2 (si $S < 100\ km$)
- $90° - \varphi'_1$ y $90° - \varphi'_2$ son arcos de la esfera de radio N_1
- $\varphi'_1 = \varphi_1$

→ $P_1 P' P_2$

$$sen\varphi'_2 = sen\varphi_1 cos\widehat{P_1P_2} + cos\varphi_1 sen\widehat{P_1P_2}\, cos\alpha_1$$

$$\varphi'_2 = \varphi_1 + \Delta\varphi' \text{ y } \widehat{P_1P_2} = S_{12}/N_1$$

$$sen(\varphi_1 + \Delta\varphi') = sen\varphi_1 cos\frac{S_{12}}{N_1} + cos\varphi_1 sen\frac{S_{12}}{N_1} cos\alpha_1$$

Desarrollando en serie

$$\Delta\varphi' = \frac{S_{12}}{N_1}cos\alpha_1 - \frac{{S_{12}}^2}{2{N_1}^2}tg\varphi_1 - \frac{{S_{12}}^3}{6{N_1}^3}cos\alpha_1 + \frac{\Delta\varphi'^2}{2}tg\varphi_1 + \frac{\Delta\varphi'^3}{6}$$

$$\Delta\varphi' = \frac{S_{12}}{N_1}cos\alpha_1 - \frac{{S_{12}}^2}{2{N_1}^2}sen^2\alpha_1 tg\varphi_1 - \frac{{S_{12}}^3}{6{N_1}^3}cos\alpha_1 sen^2\alpha_1(1 + 3tg^2\varphi_1)$$

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Fórmulas de Puissant

$$N_1 \Delta\varphi' = M_m \Delta\varphi$$

$$M_m = M_1 + \left.\frac{dM}{d\varphi}\right|_1 \frac{\Delta\varphi}{2} + \ \ldots \longrightarrow M_m = M_1 + \frac{3}{2}\frac{M_1 e^2 sen\varphi_1 cos\varphi_1}{(1 - e^2 sen^2\varphi_1)}\Delta\varphi$$

$$\Delta\varphi = \delta\varphi - A\,\delta\varphi\,\Delta\varphi \qquad A = \frac{3}{2}\frac{e^2 sen\varphi_1 cos\varphi_1}{(1 - e^2 sen^2\varphi_1)}$$

$$\delta\varphi = \frac{S_{12}}{M_1}cos\alpha_1 - \frac{{S_{12}}^2}{2N_1M_1}sen^2\alpha_1 tg\varphi_1 - \frac{{S_{12}}^3}{6{N_1}^2M_1}cos\alpha_1 sen^2\alpha_1(1 + 3tg^2\varphi_1)$$

$$\Delta\varphi \cong \delta\varphi \ \rightarrow \delta\varphi\Delta\varphi \cong (\delta\varphi)^2$$

$$\Delta\varphi = S_{12}cos\alpha_1 B - {S_{12}}^2 sen^2\alpha_1 C - h\,{S_{12}}^2 sen^2\alpha_1 D - (\delta\varphi)^2 A$$

$$B = \frac{1}{M_1} \qquad C = \frac{tg\varphi_1}{2N_1M_1} \qquad D = \frac{1 + 3tg^2\varphi_1}{6{N_1}^2} \qquad h = \frac{S_{12}cos\alpha_1}{M_1}$$

$$\varphi_2 = \varphi_1 + \Delta\varphi$$

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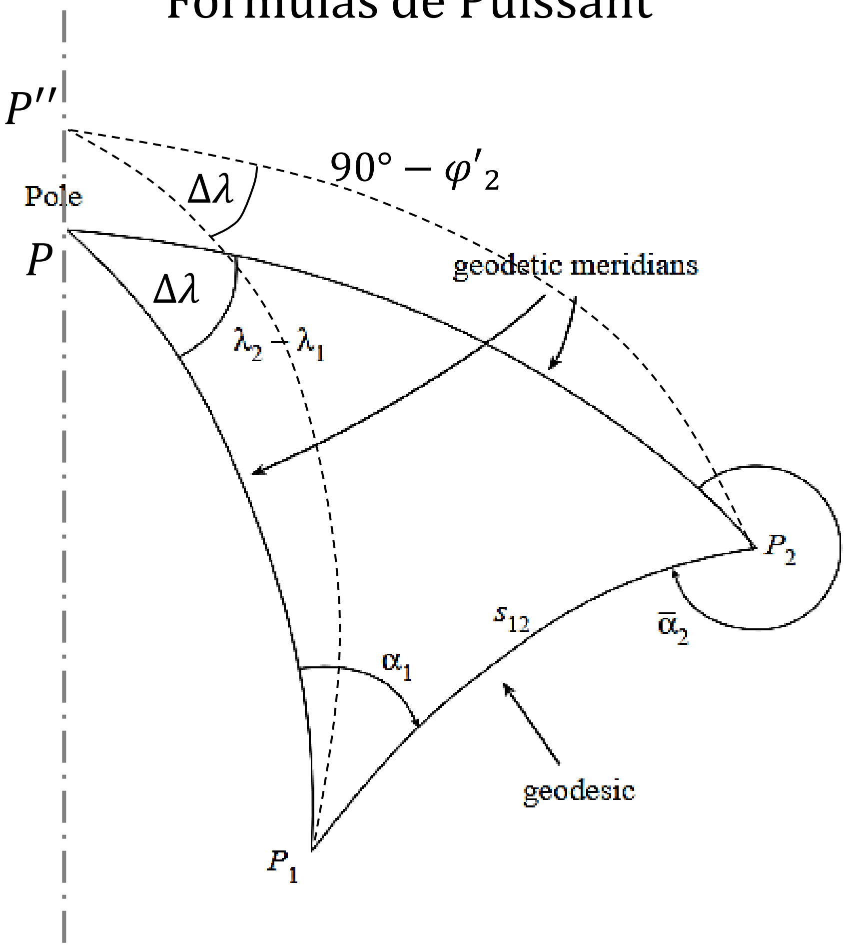
Fórmulas de Puissant



- Esfera de radio N_2, tangente en P_2
- Casi coincidente en P_1 (si $S < 100\ km$)
- $90° - \varphi'_2$ arco de la esfera de radio N_2
- $\varphi'_2 = \varphi_2$

→ $P_1 P'' P_2$

$$\frac{sen\Delta\lambda}{sen\,\frac{S_{12}}{N_2}} = \frac{sen\alpha_1}{cos\varphi_2} \rightarrow sen\Delta\lambda = sen\,\frac{S_{12}}{N_2}\frac{sen\alpha_1}{cos\varphi_2}$$

$$\Delta\lambda = \frac{S_{12}}{N_2} sen\alpha_1 sec\varphi_2 \left[1 - \frac{{S_{12}}^2}{6{N_2}^2}(1 - sen^2\alpha_1 sec^2\varphi_2)\right]$$

$$\boxed{\lambda_2 = \lambda_1 + \Delta\lambda}$$

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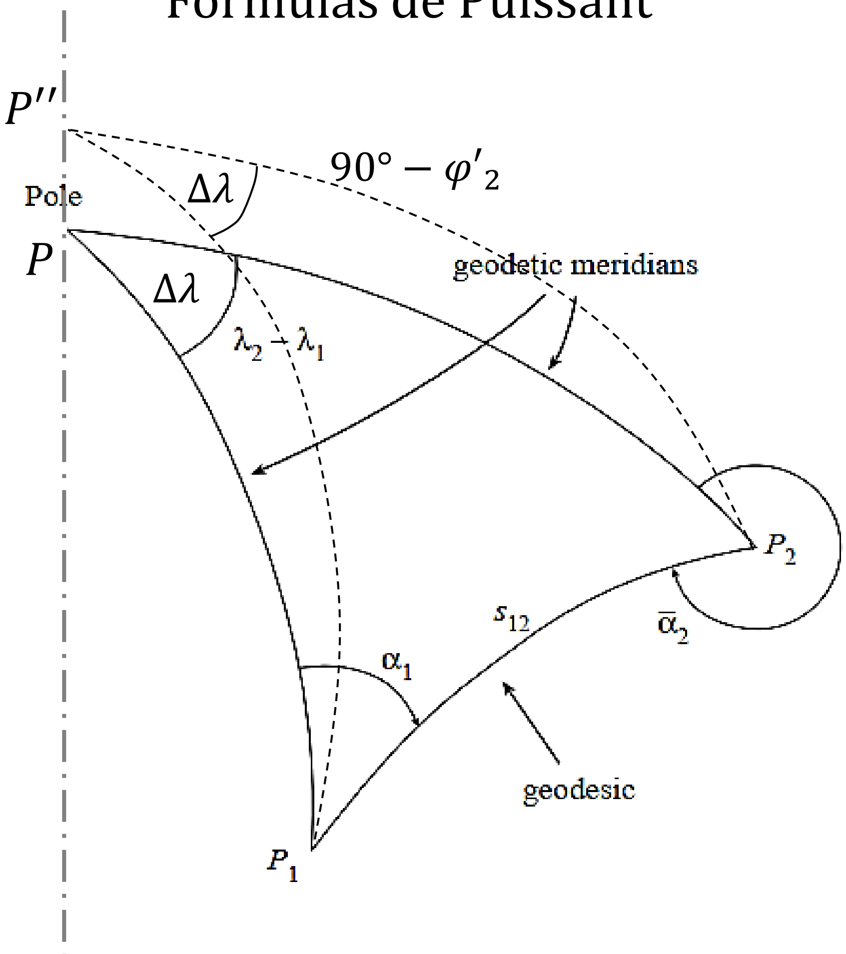
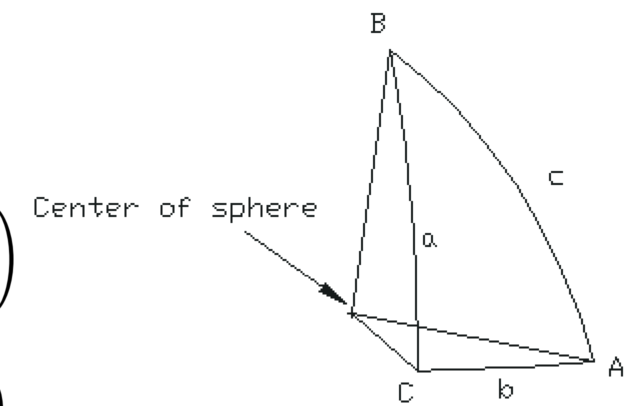
Problema Geodésico Directo[7,9,10]: → $\varphi_1, \lambda_1, \alpha_1, S_{12} \rightarrow \varphi_2, \lambda_2, \bar{\alpha}_2$

Fórmulas de Puissant



- Esfera de radio N_2, tangente en P_2
- Casi coincidente en P_1 (si $S < 100\ km$)
- $90° - \varphi'_2$ arco de la esfera de radio N_2
- $\varphi'_2 = \varphi_2$

→ $P_1P''P_2$ $\quad tg\left(\frac{A+C}{2}\right) = \frac{cos\left(\frac{a-c}{2}\right)}{cos\left(\frac{a+c}{2}\right)} cotg\left(\frac{B}{2}\right)$



$$tg\left(\frac{\alpha_1 + 360 - \bar{\alpha}_2}{2}\right) = \frac{cos\left(\frac{(90° - \varphi_2) - (90° - \varphi_1)}{2}\right)}{cos\left(\frac{(90° - \varphi_2) + (90° - \varphi_1)}{2}\right)} cotg\left(\frac{\Delta\lambda}{2}\right)$$

$$\bar{\alpha}_2 = \alpha_1 + \Delta\alpha \pm 180°$$

$$tg\left(\frac{\Delta\alpha}{2}\right) = \frac{sen\left(\frac{\varphi_1 + \varphi_2}{2}\right)}{cos\left(\frac{\Delta\varphi}{2}\right)} tg\left(\frac{\Delta\lambda}{2}\right) = \frac{sen\varphi_m}{cos\left(\frac{\Delta\varphi}{2}\right)} tg\left(\frac{\Delta\lambda}{2}\right)$$

$$\Delta\alpha = \Delta\lambda\ sen\varphi_m sec\frac{\Delta\varphi}{2} + \frac{\Delta\lambda^3}{12}\left(sen\varphi_m sec\frac{\Delta\varphi}{2} - sen^3\varphi_m sec^3\frac{\Delta\varphi}{2}\right)$$

$$\boxed{\bar{\alpha}_2 = \alpha_1 + \Delta\alpha \pm 180°}$$

(7) Rapp, Richard H. (1991). *Geometric geodesy part I*. Department of Geodetic Science and Surveying, Ohio State University.
(9) Jekeli, C. (2006). *Geometric reference systems in geodesy*. Division of Geodetic Science, School of Earth Sciences. Ohio State University.
(10) Jordan, W. (1962). *Handbook of Geodesy* (Vol. 3). Corps of Engineers, United States Army, Army Map Service

Geodesia

Problema Geodésico Inverso[7,9,10]: → $\varphi_1, \lambda_1, \varphi_2, \lambda_2 \rightarrow \alpha_1, S_{12}, \bar{\alpha}_2$

Fórmulas de Puissant

$$\Delta\lambda = \frac{S_{12}}{N_2} sen\alpha_1 sec\varphi_2 \left[1 - \frac{{S_{12}}^2}{6{N_2}^2}(1 - sen^2\alpha_1 sec^2\varphi_2)\right]$$

$$S_{12}\ sen\alpha_1 = \frac{\Delta\lambda N_2\ cos\varphi_2}{\left[1 - \frac{{S_{12}}^2}{6{N_2}^2}(1 - sen^2\alpha_1 sec^2\varphi_2)\right]} \longrightarrow S_{12}\ sen\alpha_1 = \frac{\Delta\lambda N_2\ cos\varphi_2}{1}$$

$$\Delta\varphi = S_{12} cos\alpha_1 B - {S_{12}}^2 sen^2\alpha_1 C - h\, {S_{12}}^2 sen^2\alpha_1 D - (\delta\varphi)^2 A$$

$$S_{12} cos\alpha_1 = \frac{1}{B}\left[\Delta\varphi + {S_{12}}^2 sen^2\alpha_1 C + h\, {S_{12}}^2 sen^2\alpha_1 D + (\delta\varphi)^2 A\right] \longrightarrow S_{12} cos\alpha_1 = \frac{1}{B}\left[\Delta\varphi + {S_{12}}^2 sen^2\alpha_1 C\right]$$

$$tg{\alpha^0}_1 = \frac{S_{12}\ sen\alpha_1}{S_{12} cos\alpha_1}$$

$${S^0}_{12} = \sqrt{(S_{12}\ sen\alpha_1)^2 + (S_{12} cos\alpha_1)^2}$$

$$h^0 = \frac{{S^0}_{12} cos{\alpha^0}_1}{M_1}$$

$$\Delta\alpha = \Delta\lambda\ sen\varphi_m sec\frac{\Delta\varphi}{2} + \frac{\Delta\lambda^3}{12}\left(sen\varphi_m sec\frac{\Delta\varphi}{2} - sen^3\varphi_m sec^3\frac{\Delta\varphi}{2}\right)$$

$$\bar{\alpha}_2 = \alpha_1 + \Delta\alpha \pm 180°$$

(7) Rapp, Richard H. (1991). ***Geometric geodesy part I*****. Department of Geodetic Science and Surveying, Ohio State University.**
(9) Jekeli, C. (2006). ***Geometric reference systems in geodesy*****. Division of Geodetic Science, School of Earth Sciences. Ohio State University.**
(10) Jordan, W. (1962). ***Handbook of Geodesy*** **(Vol. 3). Corps of Engineers, United States Army, Army Map Service**

Geodesia

Determinación de coordenadas(7,9,10):

Cantidades observadas en la sup. terrestre
- Direcciones
- Distancias
- Azimut astronómico

→ **Reducción** →

Elementos elipsoidales
- Direcciones
- Distancias
- Azimut geodésico

→ **Proyección al plano** →

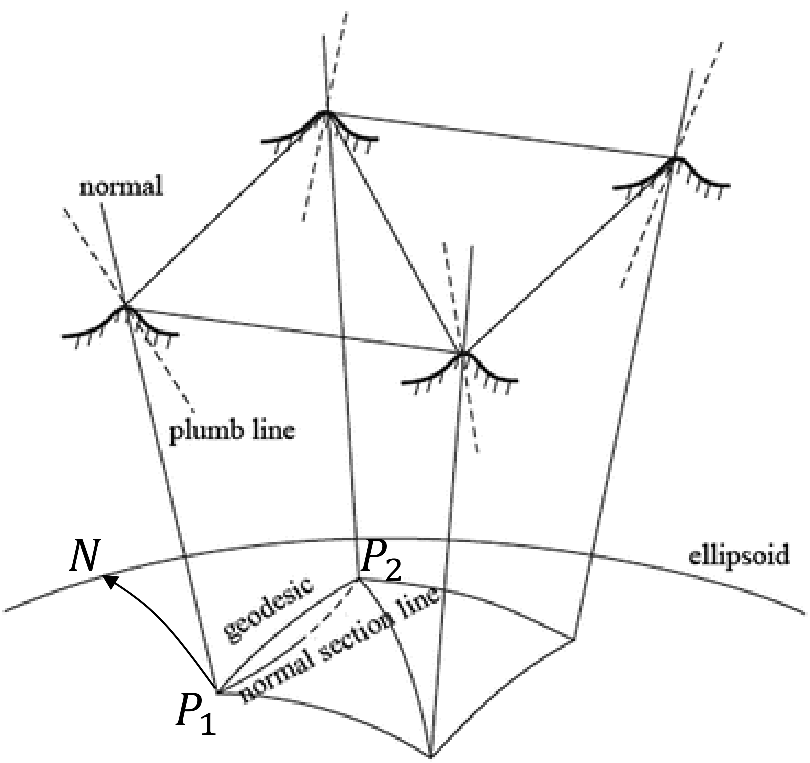
Elementos en el plano
- Direcciones
- Distancias
- Azimut de cuadrícula

Elementos elipsoidales ↓ **Compensación** ↓ Coordenadas Geodésicas (φ,λ)

Solución de triángulos elipsoidales; transporte de coordenadas

Elementos en el plano ↓ **Compensación** ↓ Coordenadas Planas (x,y)

Coordenadas Geodésicas (φ,λ) → **Proyección al plano** → Coordenadas Planas (x,y)



Fuente: Lu Z., Qu Y., Qiao S. (2014)

(7) Rapp, Richard H. (1991). *Geometric geodesy part I*. Department of Geodetic Science and Surveying, Ohio State University.
(9) Jekeli, C. (2006). *Geometric reference systems in geodesy*. Division of Geodetic Science, School of Earth Sciences. Ohio State University.
(10) Jordan, W. (1962). *Handbook of Geodesy* (Vol. 3). Corps of Engineers, United States Army, Army Map Service

Geodesia

Reducción de observaciones terrestres al elipsoide[7,9,10]:

Observaciones en la sup. terrestre →

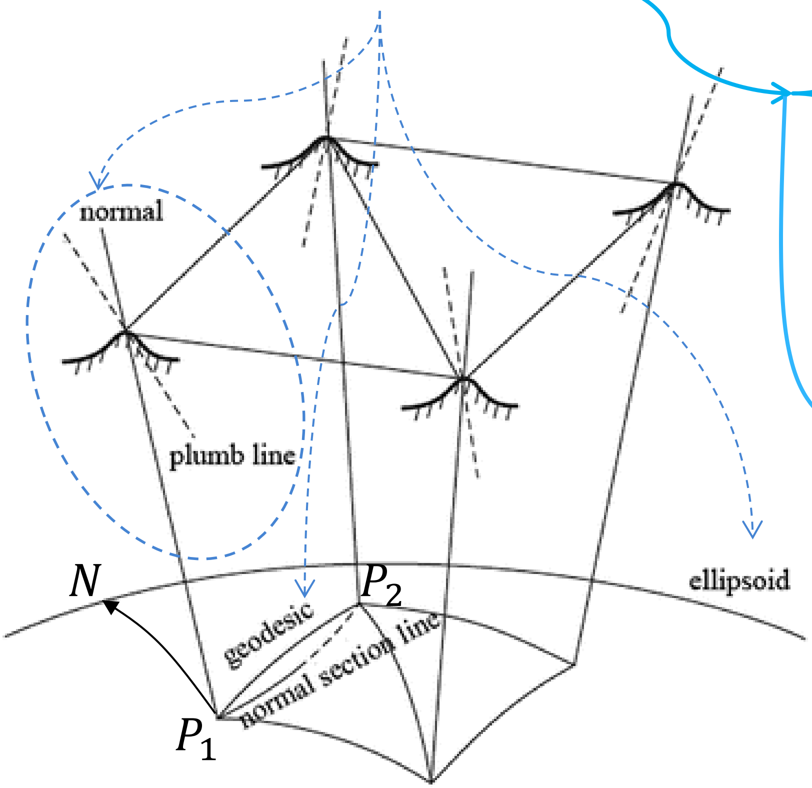
- Coordenadas horizontales de un punto
  - Direcciones
  - Distancias
  - Azimut astronómico
- Altura de un punto

Reducción → Elementos elipsoidales

Reducciones

- Corrección por desviación de la vertical;
- Corrección por alabeo de normales;
- Corrección de sección normal a geodésica;
- Reducción de distancia cenital observada;
- Reducción de distancia observada al elipsoide.

- Relación entre Latitud y Longitud Astronómica y Latitud y Longitud Geodésica;
- Relación entre Azimut Astronómico y Azimut Geodésico (Azimut de Laplace).



Fuente: Lu Z., Qu Y., Qiao S. (2014)

(7) Rapp, Richard H. (1991). *Geometric geodesy part I*. Department of Geodetic Science and Surveying, Ohio State University.
(9) Jekeli, C. (2006). *Geometric reference systems in geodesy*. Division of Geodetic Science, School of Earth Sciences. Ohio State University.
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