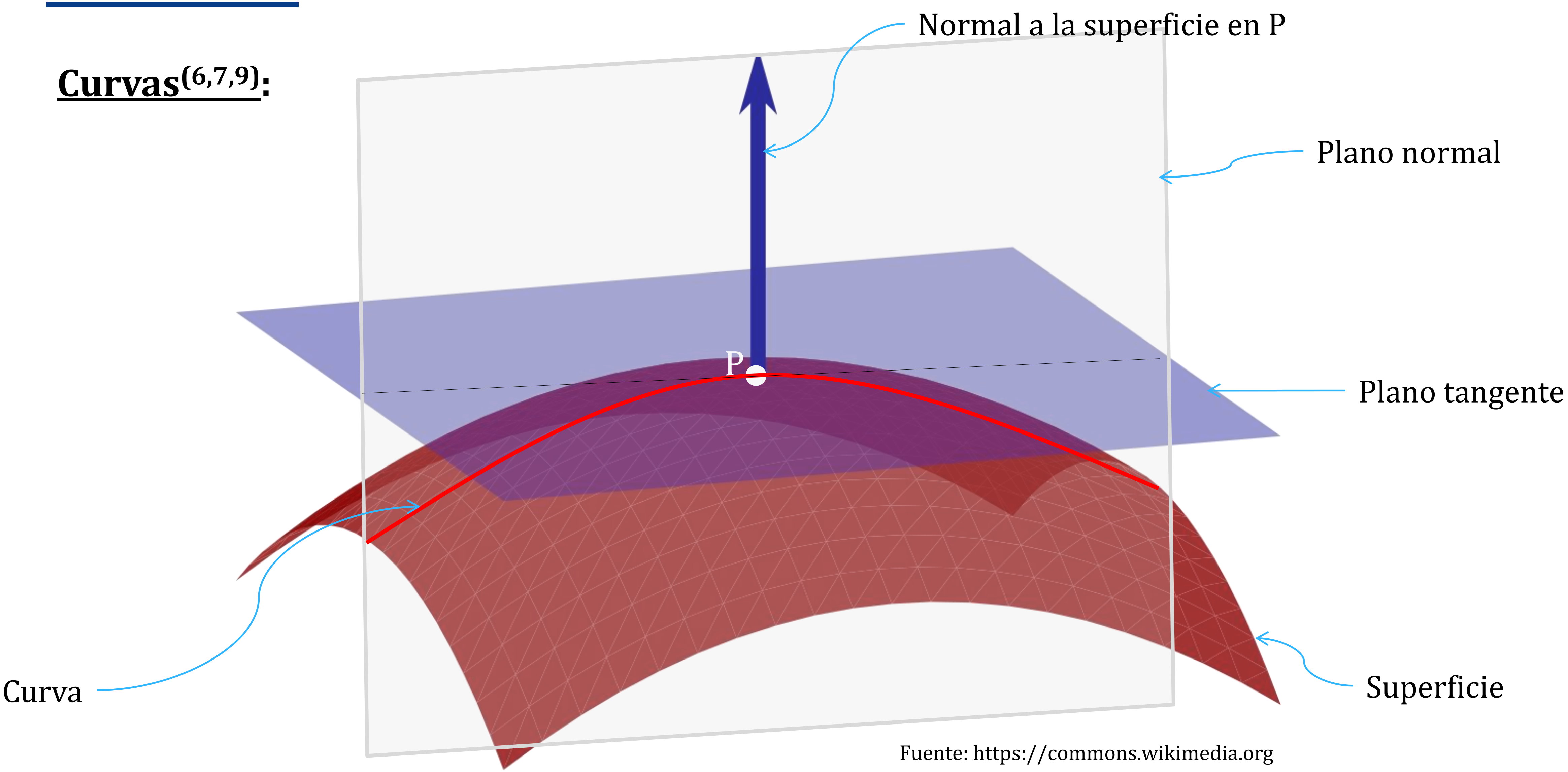
GEODESIA

Wilhelm Jordan (1842–1899)
Handbook of Geodesy

Brigadier Guy Bomford (1899–1996)
Geodesy

Geodesia

Curvas[6,7,9]:



Fuente: https://commons.wikimedia.org

Curva de sección normal: es una curva plana creada por la intersección de un plano que contiene la normal a la superficie (un plano de sección normal) con la superficie.

(6) Krakiwsky, Edward J., and Donald B. Thomson (1974). *Geodetic position computations*.. Department of Surveying Engineering, University of New Brunswick.
(7) Rapp, Richard H. (1991). *Geometric geodesy part I*. Department of Geodetic Science and Surveying, Ohio State University.
(9) Jekeli, C. (2006). *Geometric reference systems in geodesy*. Division of Geodetic Science, School of Earth Sciences. Ohio State University.

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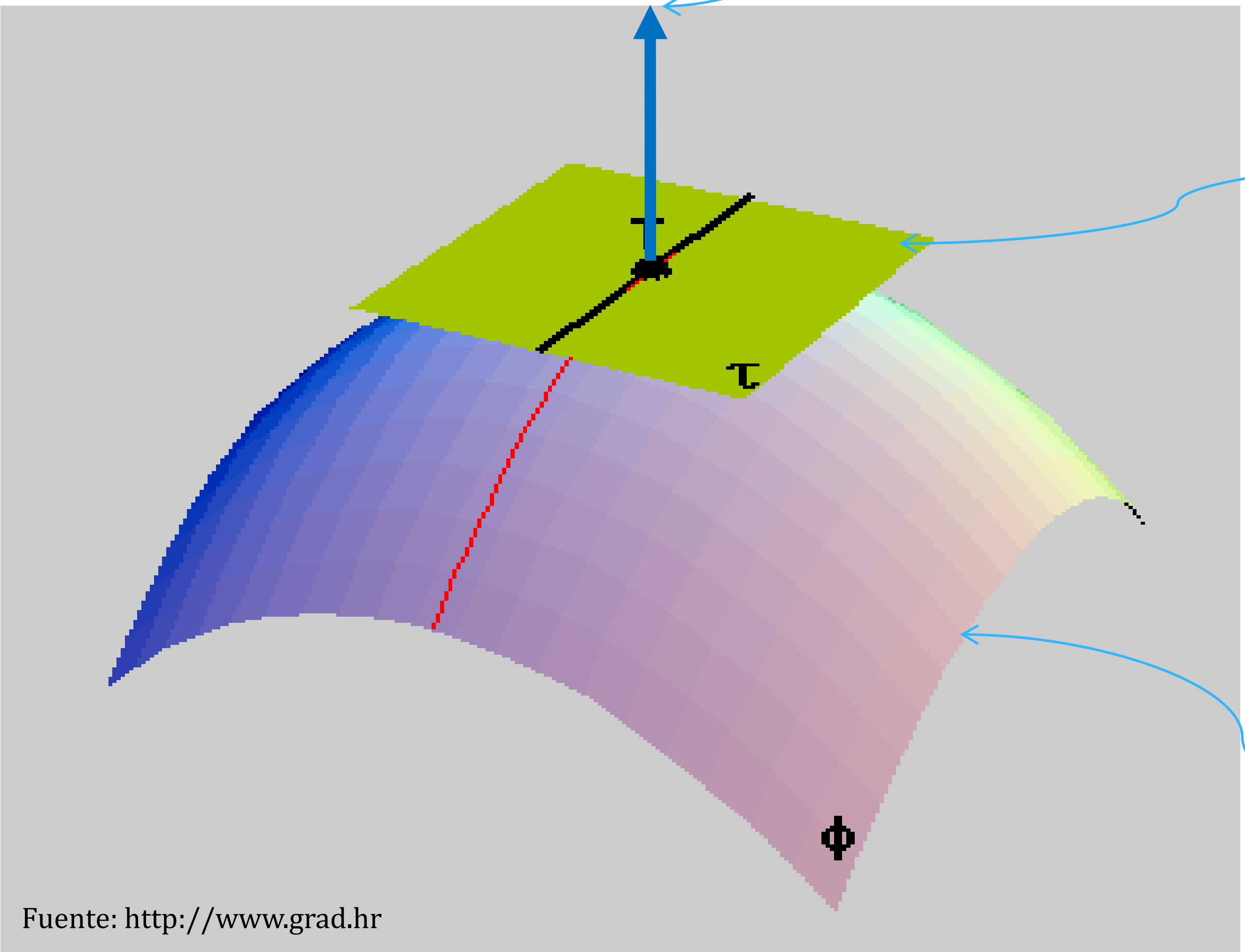
Curvas[6,7,9]:



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(6) Krakiwsky, Edward J., and Donald B. Thomson (1974). *Geodetic position computations*.. Department of Surveying Engineering, University of New Brunswick.
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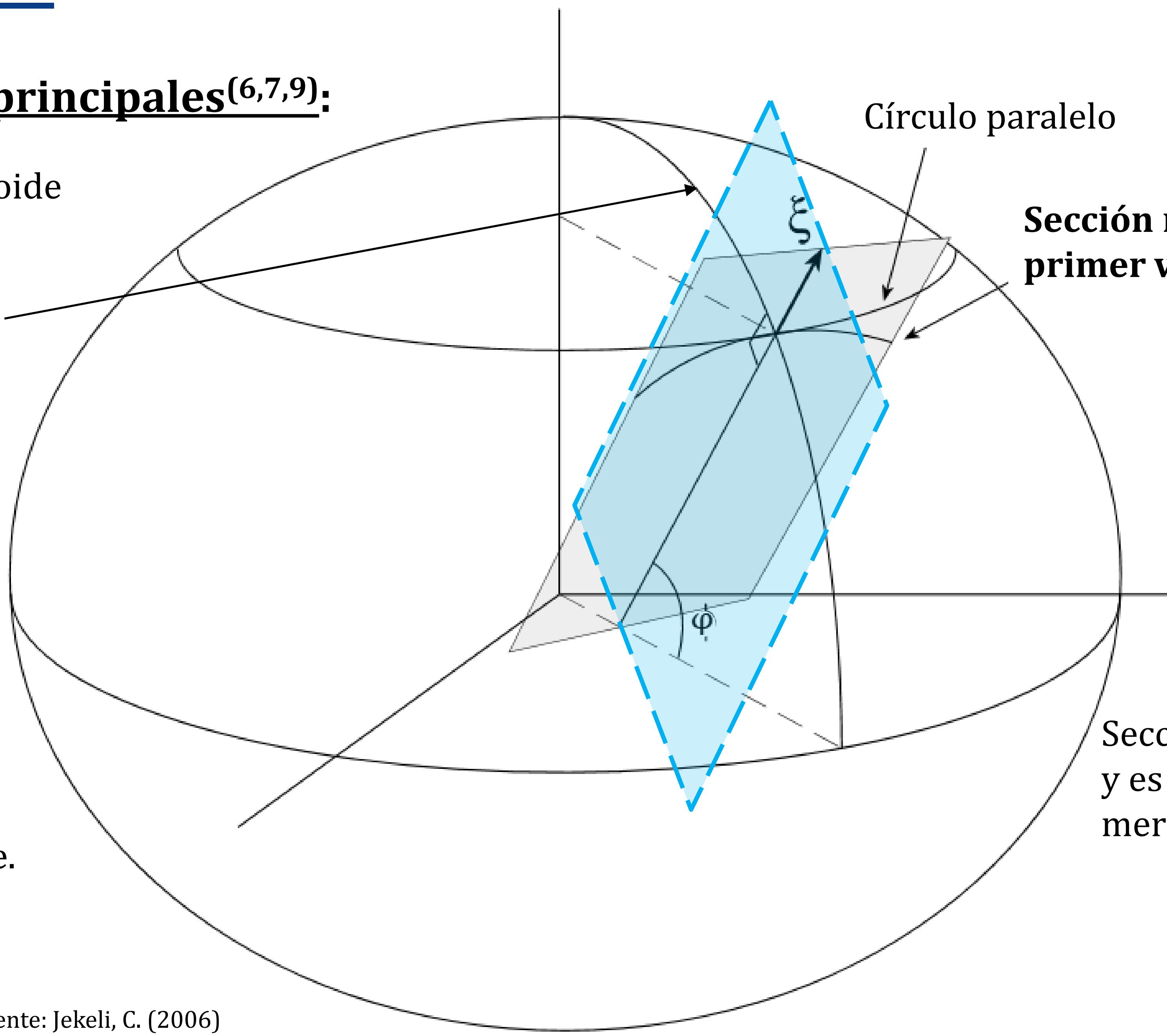
Secciones principales[6,7,9]:

ξ: Normal al elipsoide



Sección normal a la meridiana (ó primer vertical)

Sección que pasa por un punto y es perpendicular a la sección meridiana.

Sección meridiana

Sección que pasa por un punto y contiene a los polos del elipsoide.

Fuente: Jekeli, C. (2006)

(6) Krakiwsky, Edward J., and Donald B. Thomson (1974). *Geodetic position computations*.. Department of Surveying Engineering, University of New Brunswick.
(7) Rapp, Richard H. (1991). *Geometric geodesy part I*. Department of Geodetic Science and Surveying, Ohio State University.
(9) Jekeli, C. (2006). *Geometric reference systems in geodesy*. Division of Geodetic Science, School of Earth Sciences. Ohio State University.

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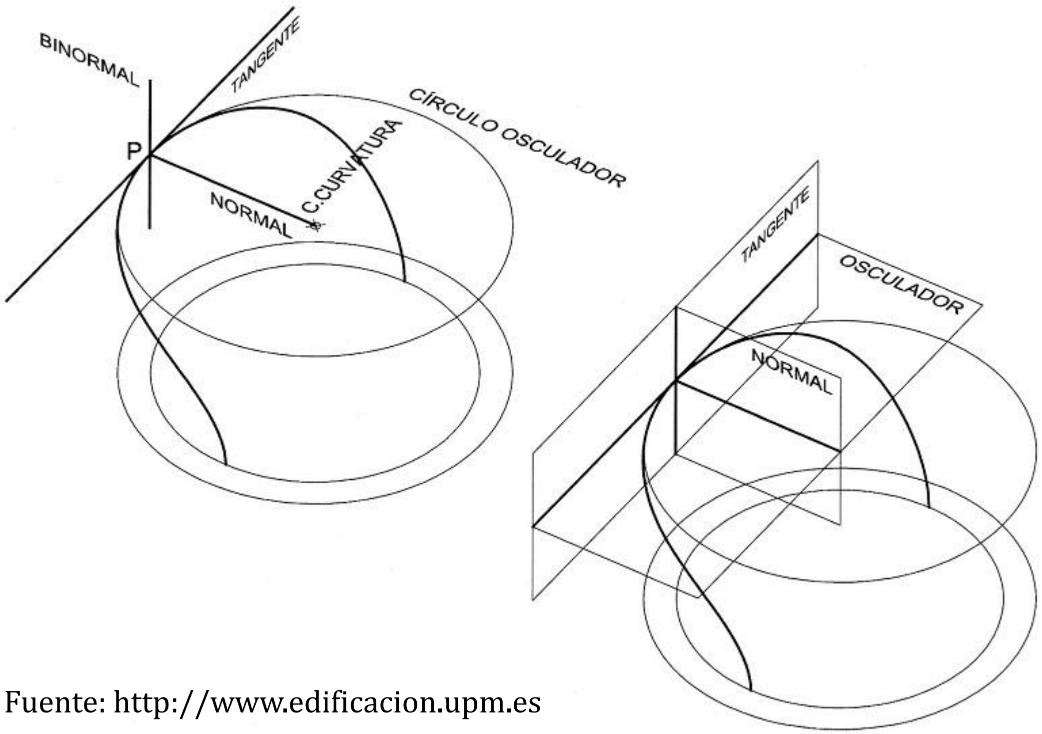
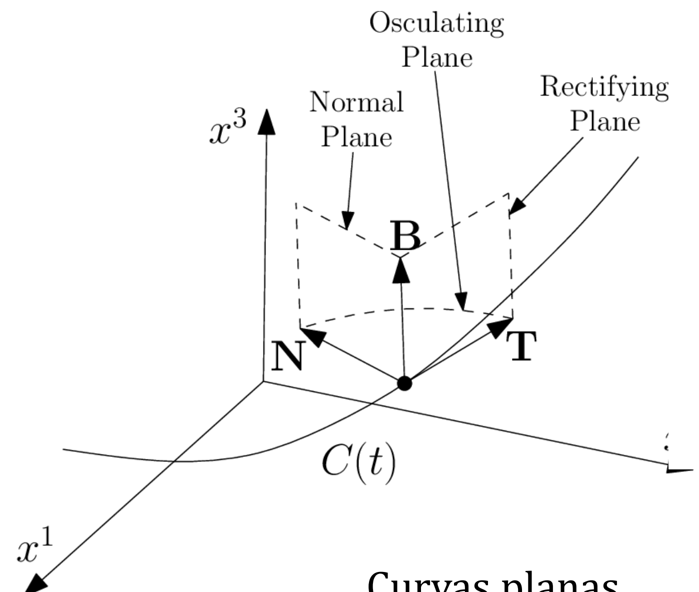
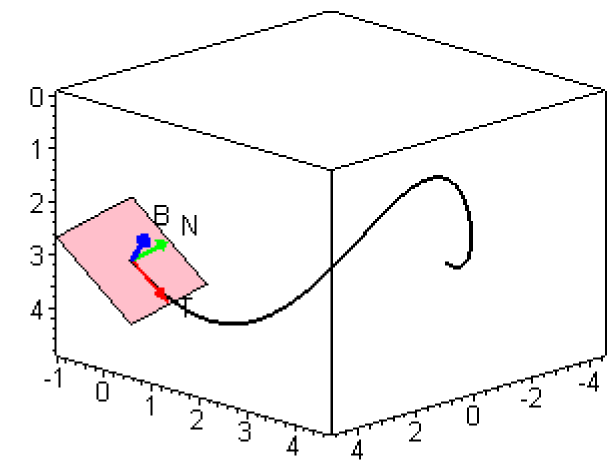
Triedo de Frenet:



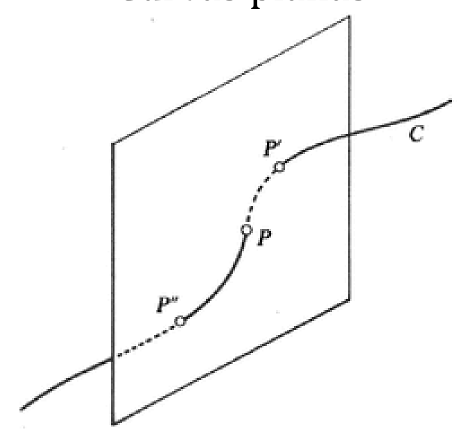
Fuente: http://www.edificacion.upm.es



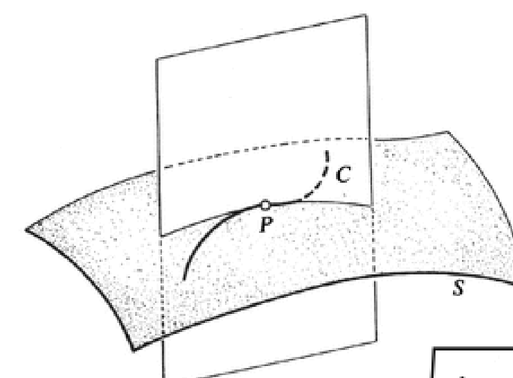


Fuente: http://faculty.etsu.edu

Curvas planas



Doble curvatura



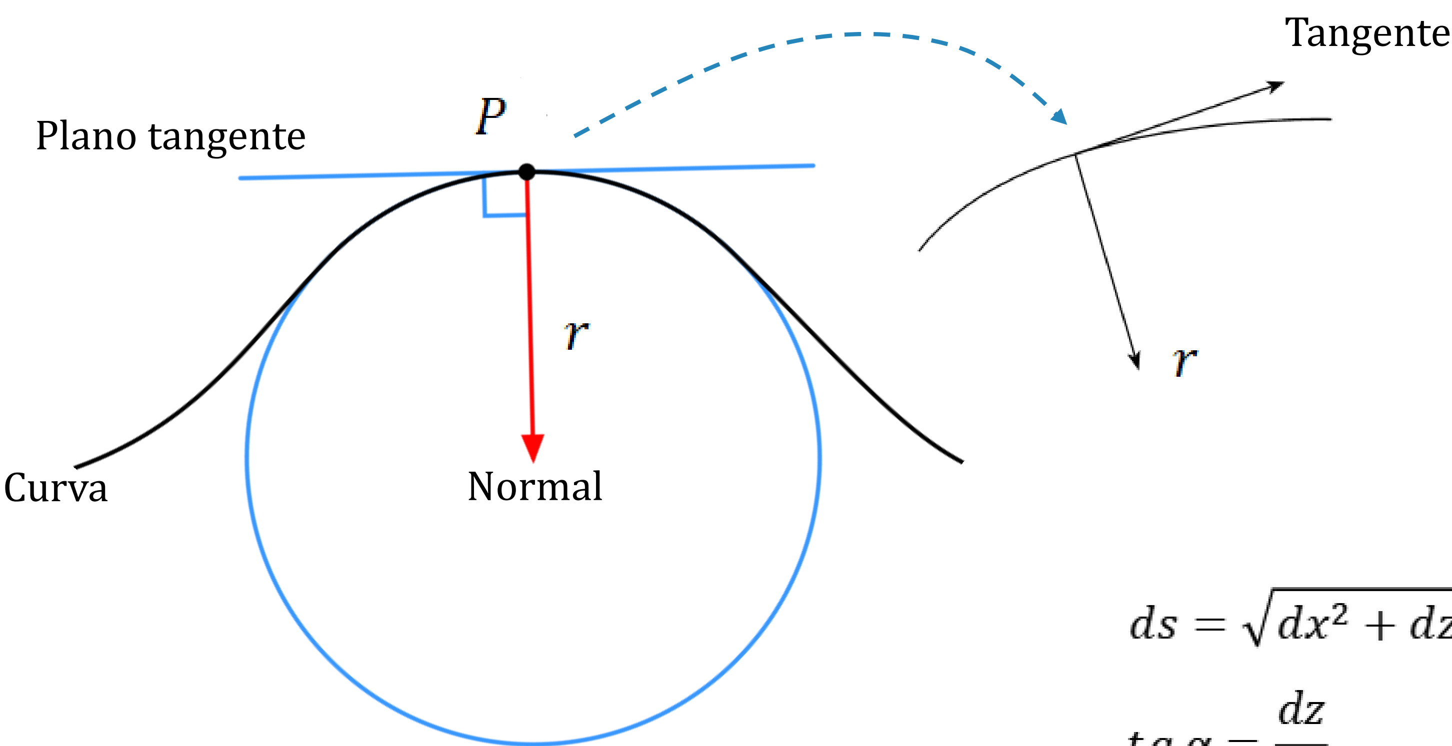
$$\sqrt{\frac{1}{\chi^2} + \frac{1}{\tau^2}}$$

Geodesia

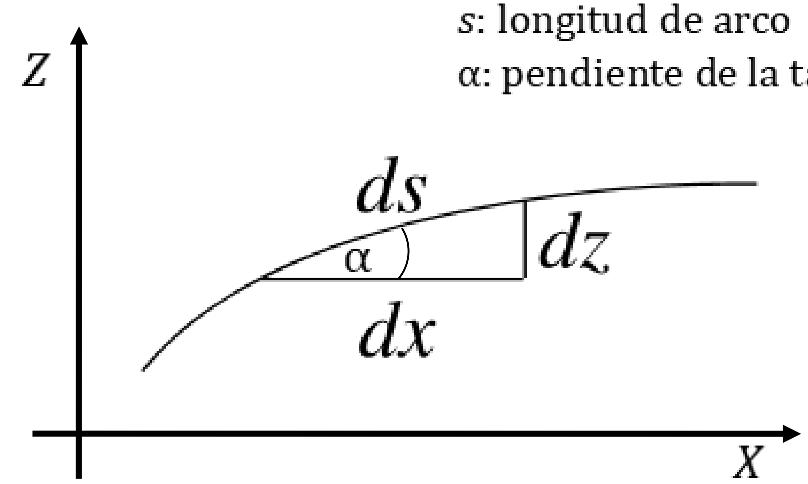
Curvatura[6,7,9]: tasa de cambio absoluta del ángulo de la tangente a la curva respecto a un elemento de longitud de arco de la curva.

$$\chi = \left|\frac{d\alpha}{ds}\right|$$

s: longitud de arco
α: pendiente de la tangente



Fuente: https://commons.wikimedia.org
https://solitaryroad.com/c361.html



$$ds = \sqrt{dx^2 + dz^2}$$

$$tg\,\alpha = \frac{dz}{dx}$$

$$\chi = \frac{\left|\frac{d^2z}{dx^2}\right|}{\left(1 + \left(\frac{dz}{dx}\right)^2\right)^{3/2}}$$

Radio de Curvatura: inversa de la curvatura. $\rho = \frac{1}{\chi}$

Curvatura: “es una medida de cuánto se aparta una curva de ser recta”

(6) Krakiwsky, Edward J., and Donald B. Thomson (1974). *Geodetic position computations*.. Department of Surveying Engineering, University of New Brunswick.
(7) Rapp, Richard H. (1991). *Geometric geodesy part I*. Department of Geodetic Science and Surveying, Ohio State University.
(9) Jekeli, C. (2006). *Geometric reference systems in geodesy*. Division of Geodetic Science, School of Earth Sciences. Ohio State University.

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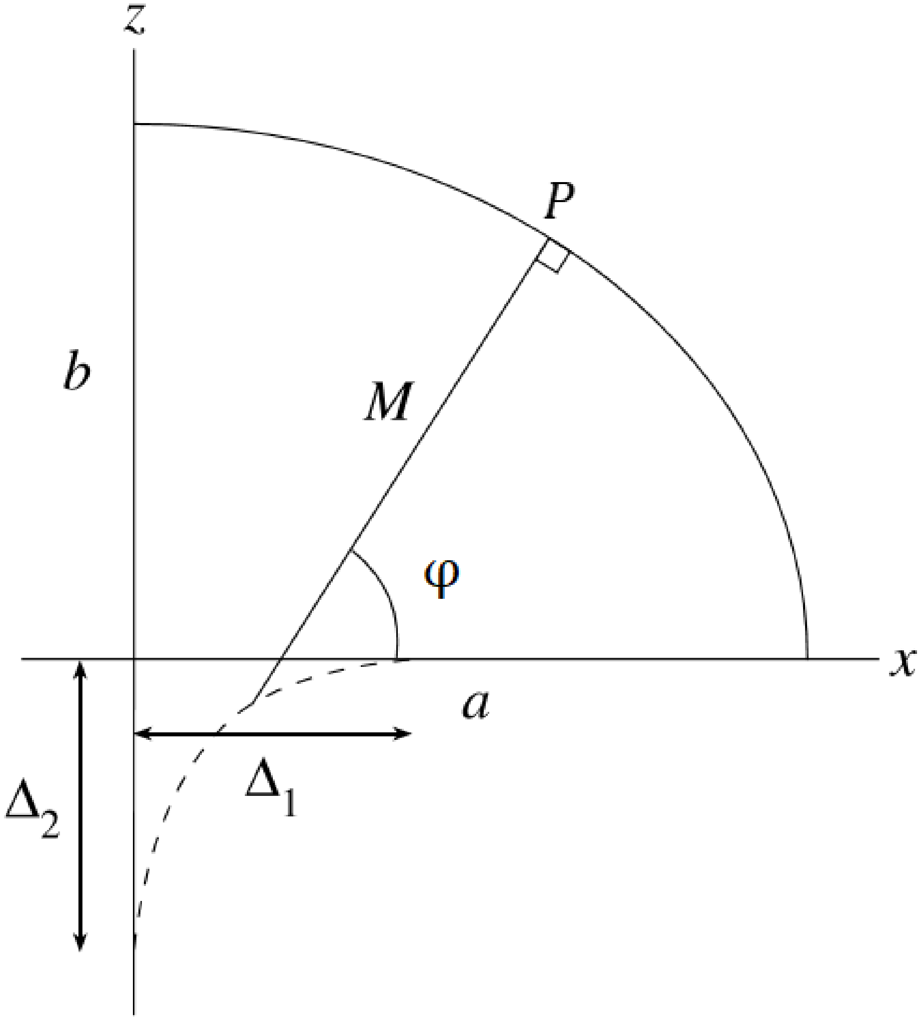
Radio de curvatura[6,7,9]**:**

Sección Meridiana



Fuente: Jekeli, C. (2006)

$$\chi = \frac{\left|\frac{d^2z}{dx^2}\right|}{\left(1+\left(\frac{dz}{dx}\right)^2\right)^{3/2}}$$

$$\frac{dz}{dx} = -\frac{a^2}{b^2}\frac{x}{z} = -\, cotg\,\varphi$$

$$\left\{ \begin{aligned} \chi &= \frac{a}{b^2}(1-e^2sen^2\varphi)^{3/2} \\ \frac{1}{\chi} &= \rho = \frac{a(1-e^2)}{(1-e^2sen^2\varphi)^{3/2}} \end{aligned} \right.$$

$$\chi = \left|\frac{d\alpha}{ds}\right| \rightarrow \frac{1}{M} = \left|\frac{d\varphi}{ds}\right| \Longrightarrow ds_{meridiana} = Md\varphi$$

$$\begin{aligned} \varphi_{ecuador} &= ? \Longrightarrow \\ \varphi_{polo} &= ? \Longrightarrow \end{aligned} \left\{ \begin{aligned} M_{ecuador} &= \frac{a(1-e^2)}{(1-e^2sen^2\varphi)^{3/2}} = ? \\ M_{polo} &= \frac{a(1-e^2)}{(1-e^2sen^2\varphi)^{3/2}} = ? \end{aligned} \right.$$

(6) Krakiwsky, Edward J., and Donald B. Thomson (1974). *Geodetic position computations*.. Department of Surveying Engineering, University of New Brunswick.

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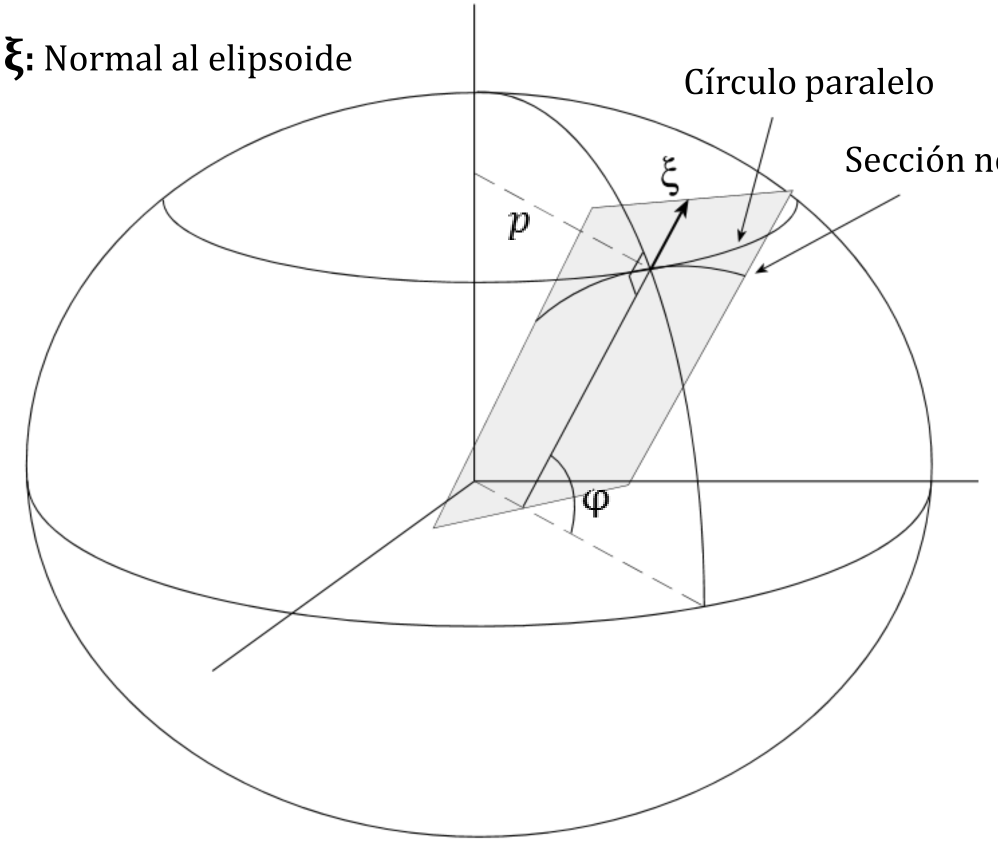
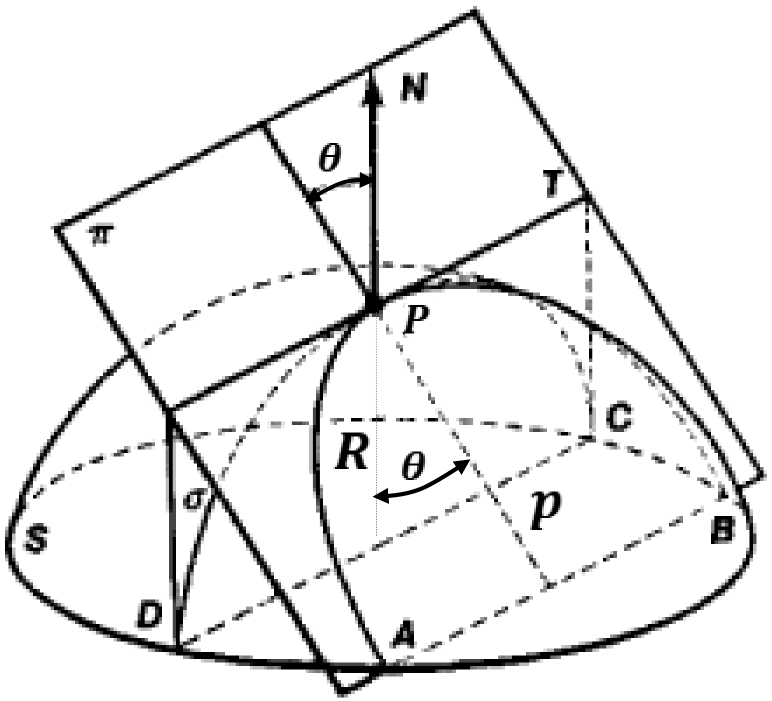
Geodesia

<u>Radio de curvatura</u>[6,7,9]:
Sección Normal a la Meridiana



Fuente: Jekeli, C. (2006)

<u>Meusnier</u>: el radio de curvatura de una sección inclinada es igual al radio de curvatura de una sección normal multiplicada por el coseno del ángulo entre ambas.



Fuente: https://encyclopedia2.thefreedictionary.com

$$p = R\cos\theta \iff \frac{1}{p}\cos\theta = \frac{1}{R}$$

(6) Krakiwsky, Edward J., and Donald B. Thomson (1974). *Geodetic position computations*.. Department of Surveying Engineering, University of New Brunswick.
(7) Rapp, Richard H. (1991). *Geometric geodesy part I*. Department of Geodetic Science and Surveying, Ohio State University.
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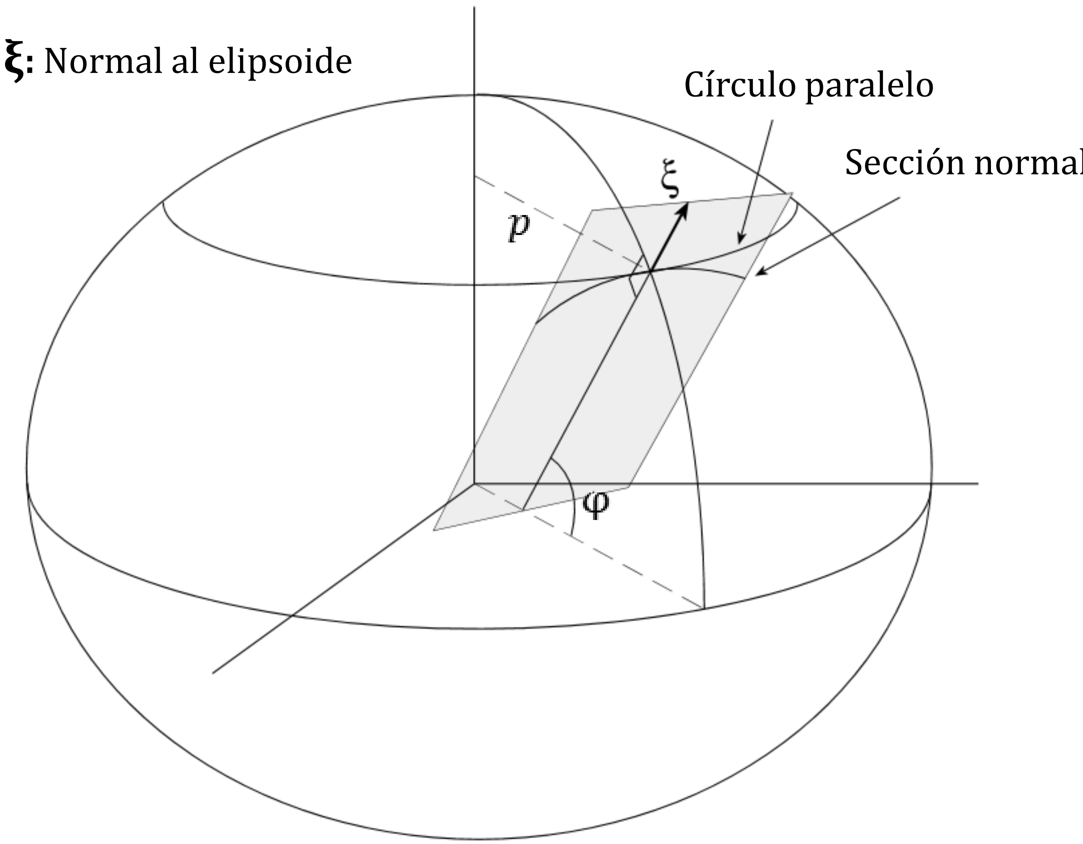
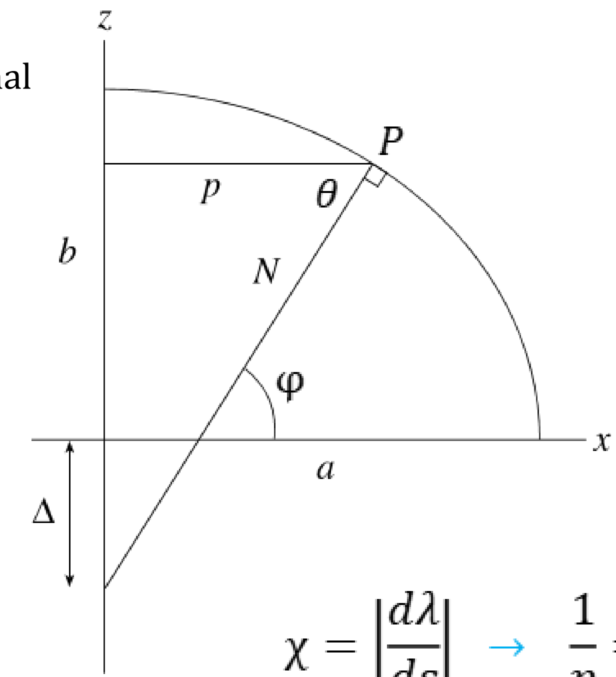
Radio de curvatura[6,7,9]:

Sección Normal a la Meridiana



Fuente: Jekeli, C. (2006)

Meusnier: el radio de curvatura de una sección inclinada es igual al radio de curvatura de una sección normal multiplicada por el coseno del ángulo entre ambas.



$$\frac{1}{p}\cos\theta = \frac{1}{R} \rightarrow \frac{1}{p}\cos\varphi = \frac{1}{N}$$

$$\begin{cases} p = N\cos\varphi \\ x = \dfrac{a\cos\varphi}{(1-e^2 sen^2\varphi)^{1/2}} \end{cases}$$

$$N = \frac{a}{(1-e^2 sen^2\varphi)^{1/2}}$$

$$\chi = \left|\frac{d\lambda}{ds}\right| \rightarrow \frac{1}{p} = \left|\frac{d\lambda}{ds}\right| \Longrightarrow ds_{paralelo} = N\cos\varphi\, d\lambda$$

$$\varphi_{ecuador} = ? \Longrightarrow \begin{cases} N_{ecuador} = \dfrac{a}{(1-e^2 sen^2\varphi)^{1/2}} = ? \\ N_{polo} = \dfrac{a}{(1-e^2 sen^2\varphi)^{1/2}} = ? \end{cases}$$

$$\varphi_{polo} = ? \Longrightarrow$$

(6) Krakiwsky, Edward J., and Donald B. Thomson (1974). *Geodetic position computations*.. Department of Surveying Engineering, University of New Brunswick.
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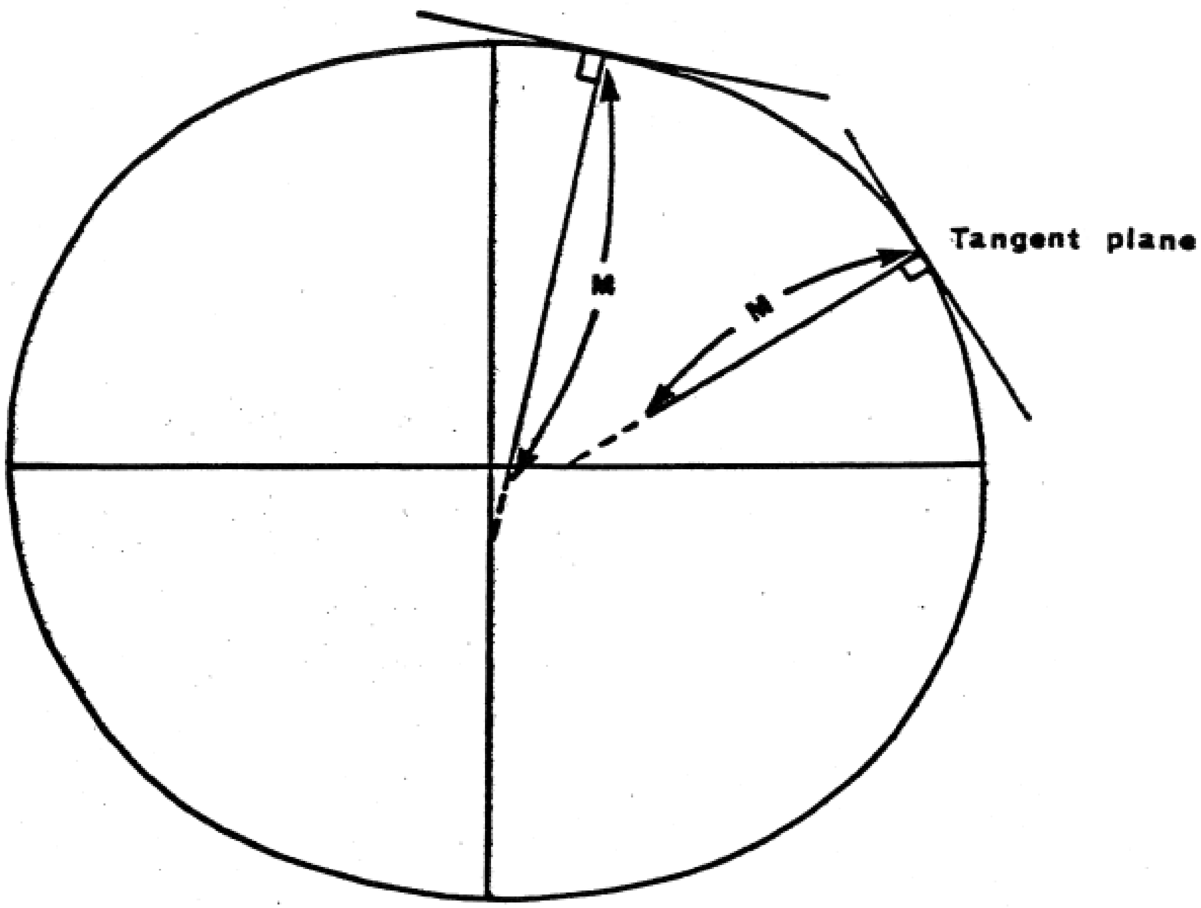
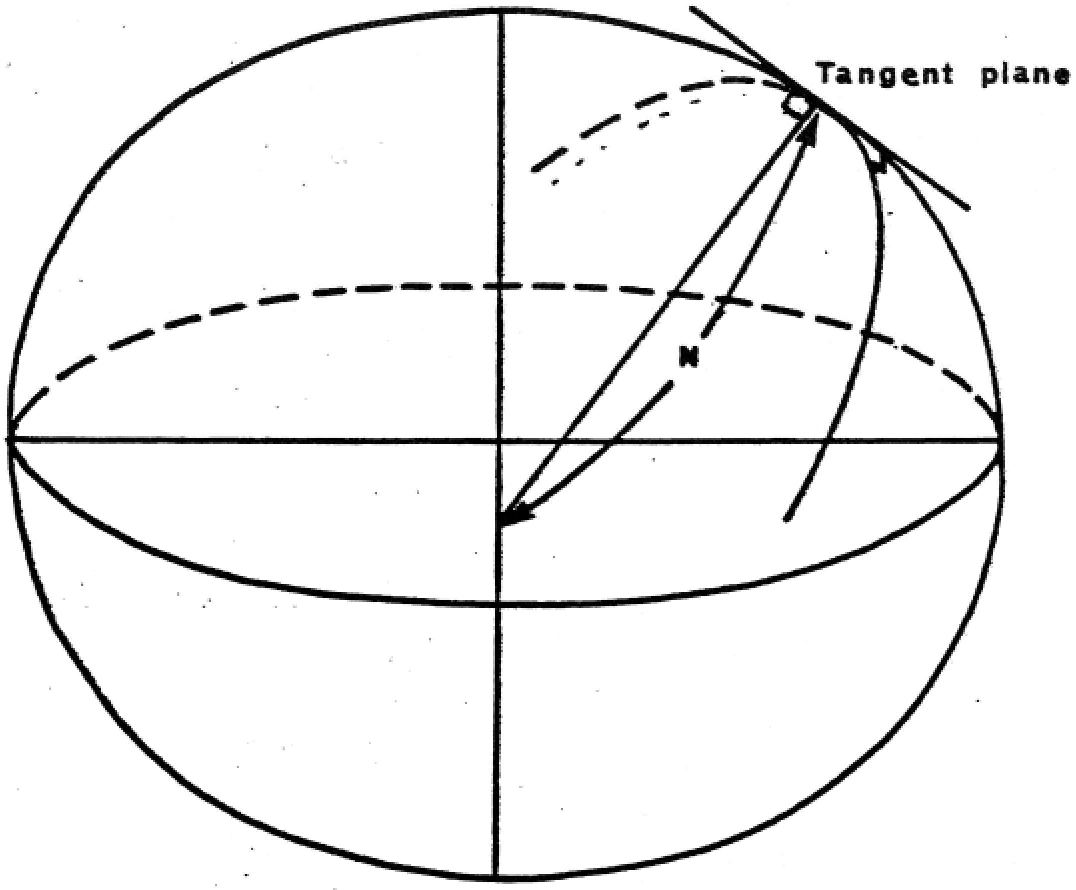
Geodesia

Secciones principales[6,7,9]:





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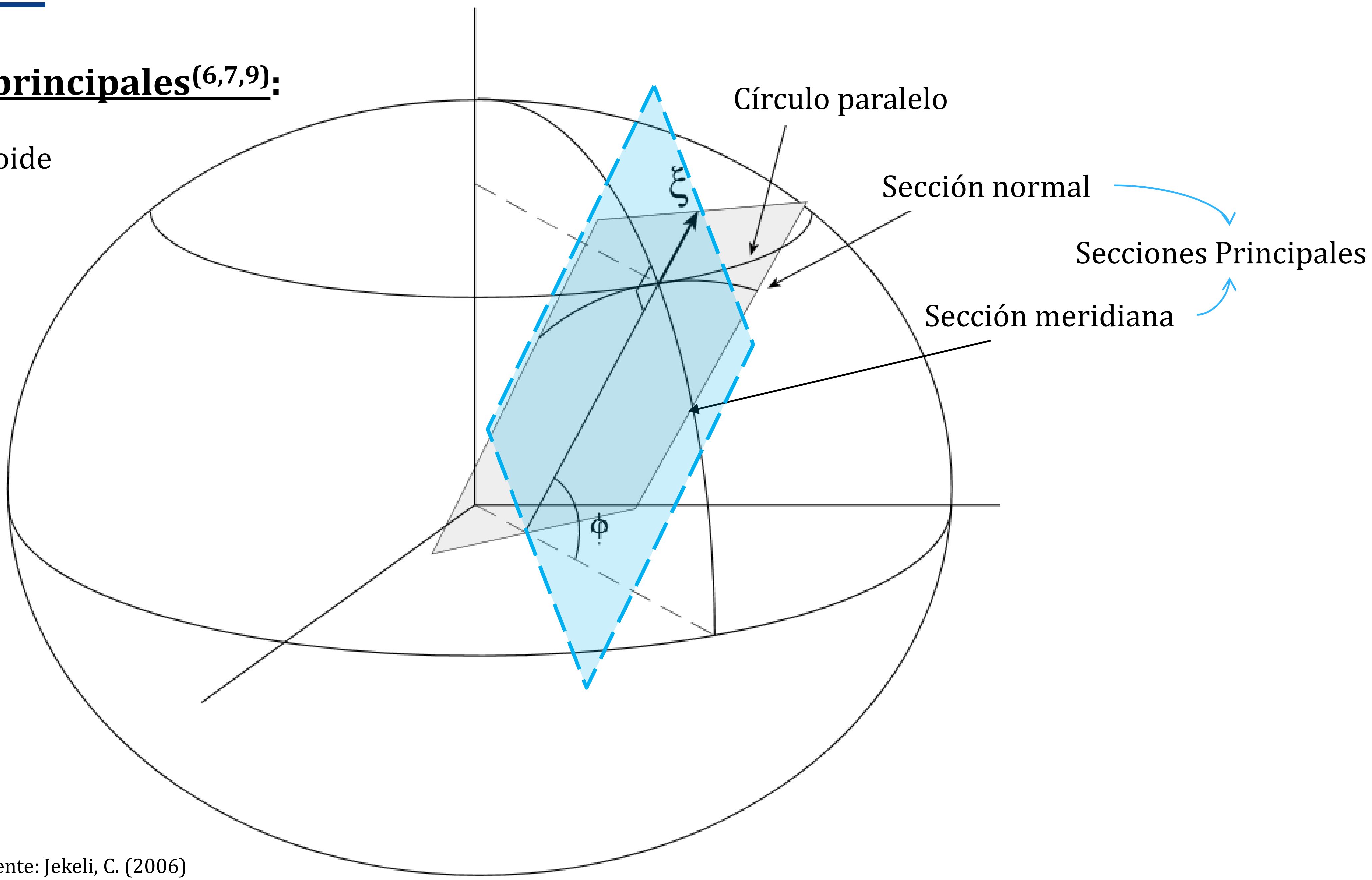
Geodesia

Secciones principales(6,7,9):

ξ: Normal al elipsoide



Fuente: Jekeli, C. (2006)

(6) Krakiwsky, Edward J., and Donald B. Thomson (1974). *Geodetic position computations*.. Department of Surveying Engineering, University of New Brunswick.
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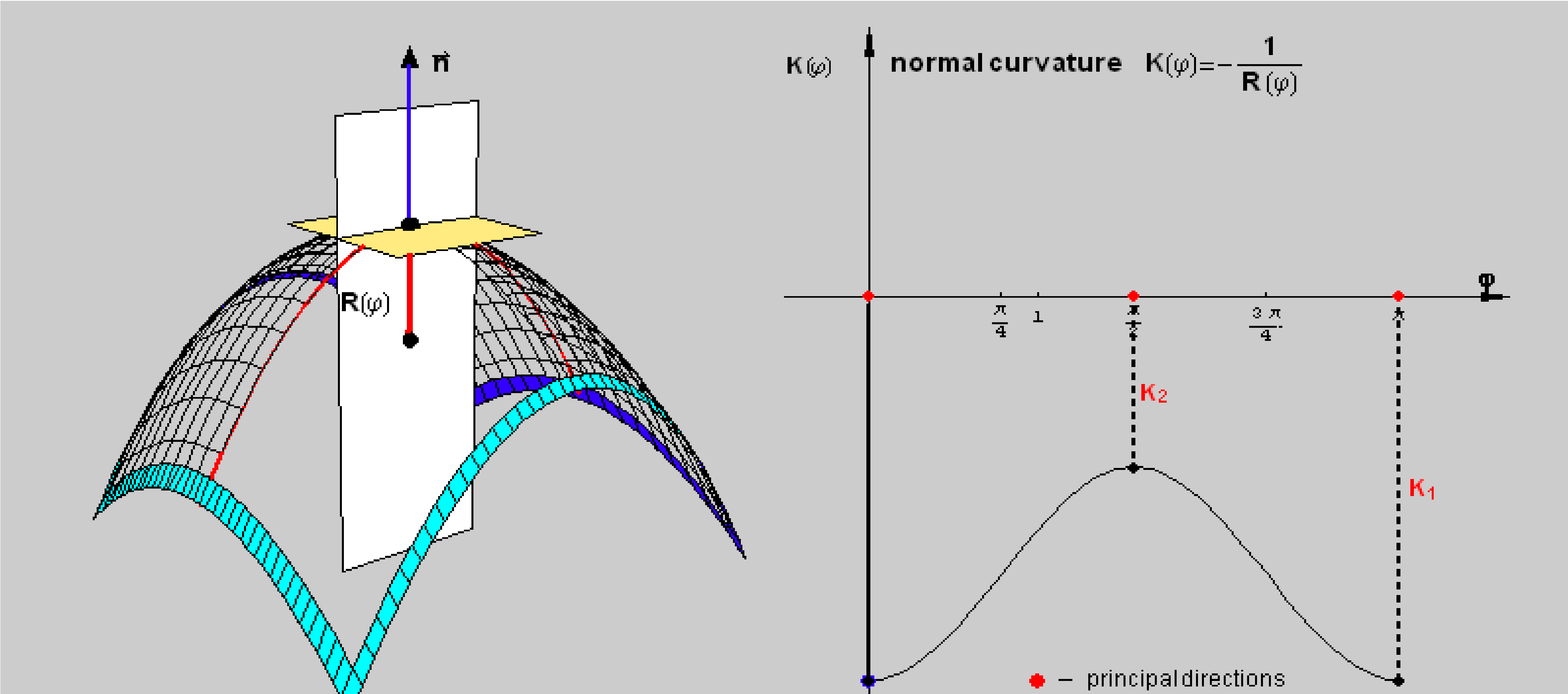
Curvatura y Radio de Curvatura [6,7,9]:



Fuente: http://www.grad.hr

(6) Krakiwsky, Edward J., and Donald B. Thomson (1974). *Geodetic position computations*.. Department of Surveying Engineering, University of New Brunswick.
(7) Rapp, Richard H. (1991). *Geometric geodesy part I*. Department of Geodetic Science and Surveying, Ohio State University.
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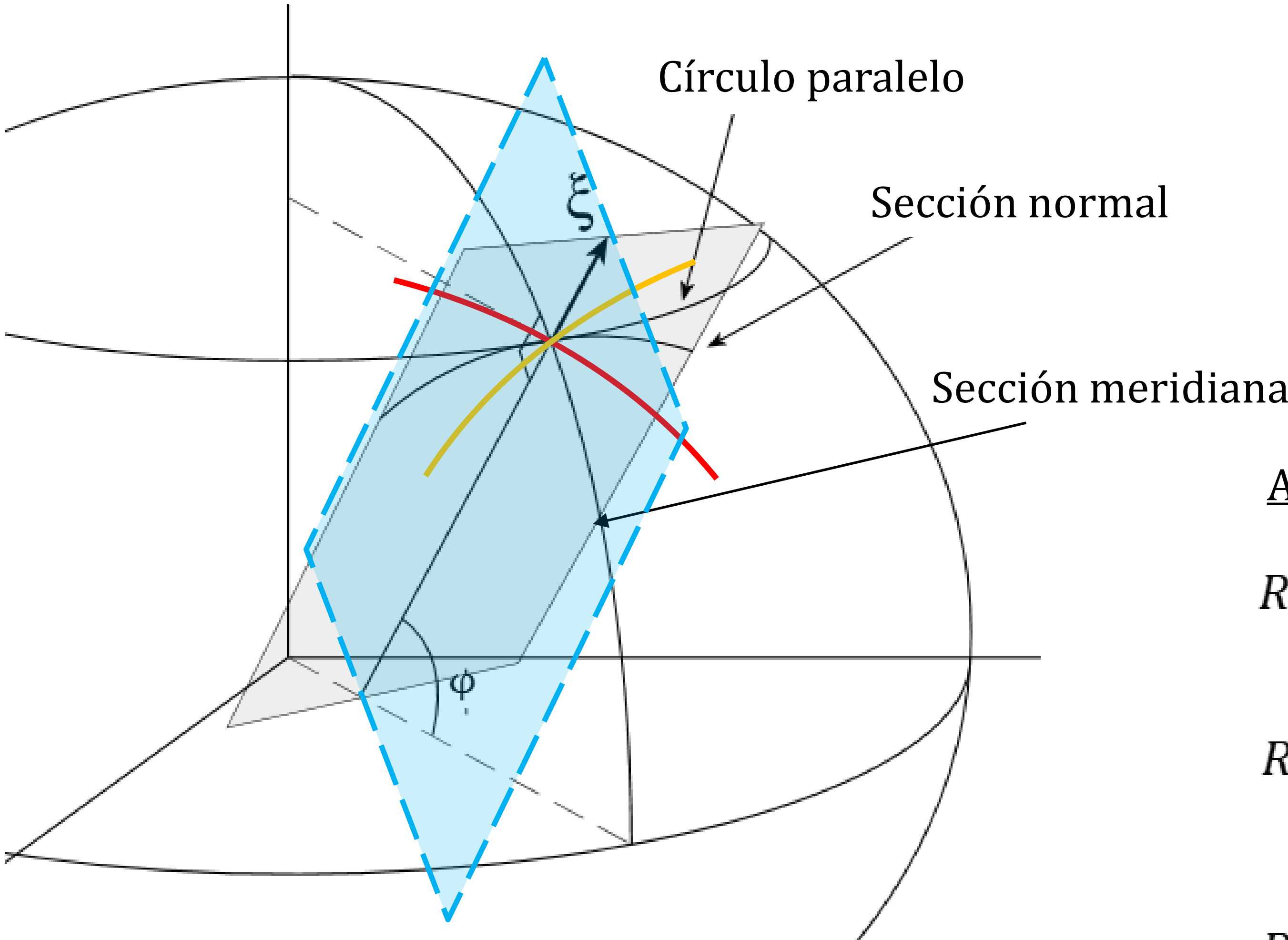
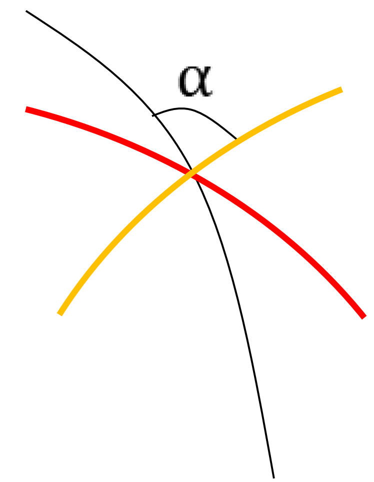
Curvas en el elipsoide[6,7,9]:

ξ: Normal al elipsoide



Fuente: Jekeli, C. (2006)

Teorema de Euler: curvatura de una sección de acimut α en función de los radio de curvaturas de las secciones principales.



$$\chi_{\alpha} = \frac{1}{R_{\alpha}} = \frac{sen^2\alpha}{N} + \frac{cos^2\alpha}{M}$$

Aproximaciones

$$R_G = \frac{1}{2\pi}\int_0^{2\pi} R_{\alpha} d\alpha = \int_0^{2\pi} \frac{d\alpha}{\frac{sen^2\alpha}{N} + \frac{cos^2\alpha}{M}} = \sqrt{MN}$$

$$R_m = \frac{1}{\frac{1}{2}\left(\frac{1}{N} + \frac{1}{M}\right)}$$

$$R = \frac{1}{3}(a + a + b) \rightarrow R = a\left(1 - \frac{e^2}{6} - \frac{e^4}{24} - \frac{e^6}{48} - \ldots\right)$$

(6) Krakiwsky, Edward J., and Donald B. Thomson (1974). *Geodetic position computations*.. Department of Surveying Engineering, University of New Brunswick.
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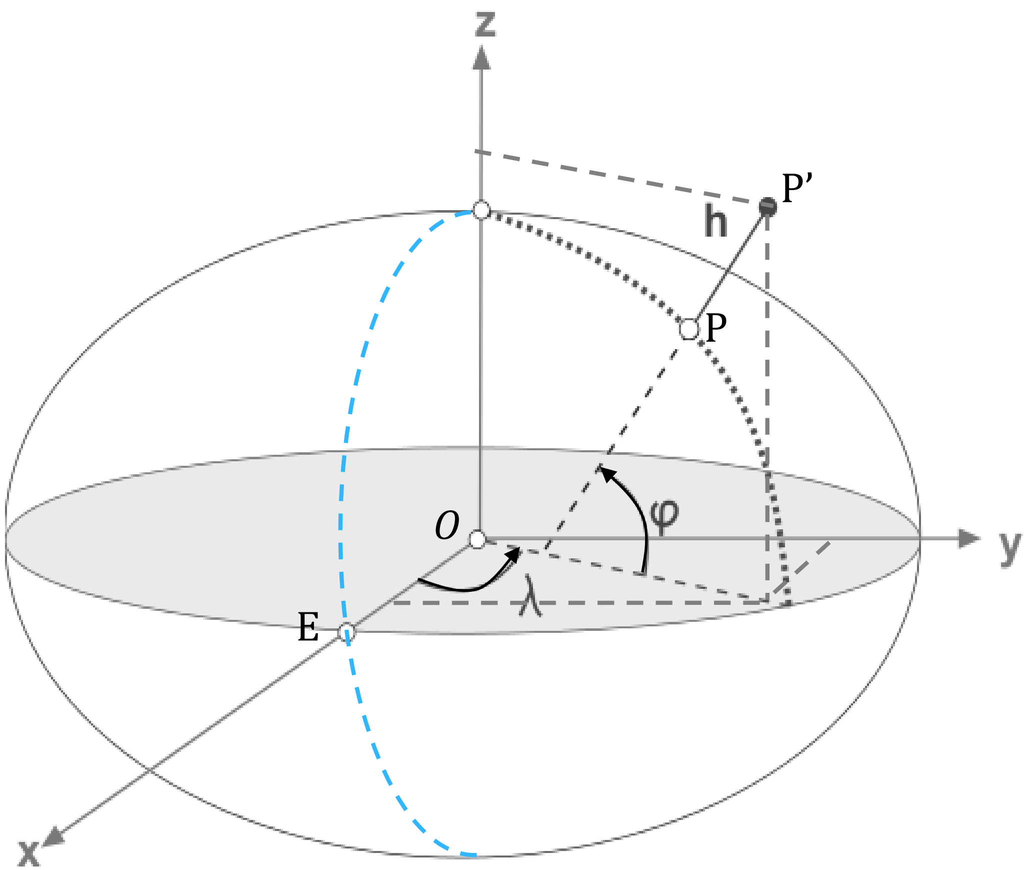
Geodesia

Coordenadas Cartesianas Rectangulares[6,7,9] asociadas al elipsoide de referencia



Fuente: https://gssc.esa.int
ESA: European Space Agency

OX
OY
OZ
Sistema de la mano derecha

Ecuaciones de transformación

φ
λ
Coordenadas curvilíneas en la superficie del elipsoide

h: Altura sobre el elipsoide

(6) Krakiwsky, Edward J., and Donald B. Thomson (1974). *Geodetic position computations*.. Department of Surveying Engineering, University of New Brunswick.
(7) Rapp, Richard H. (1991). *Geometric geodesy part I*. Department of Geodetic Science and Surveying, Ohio State University.
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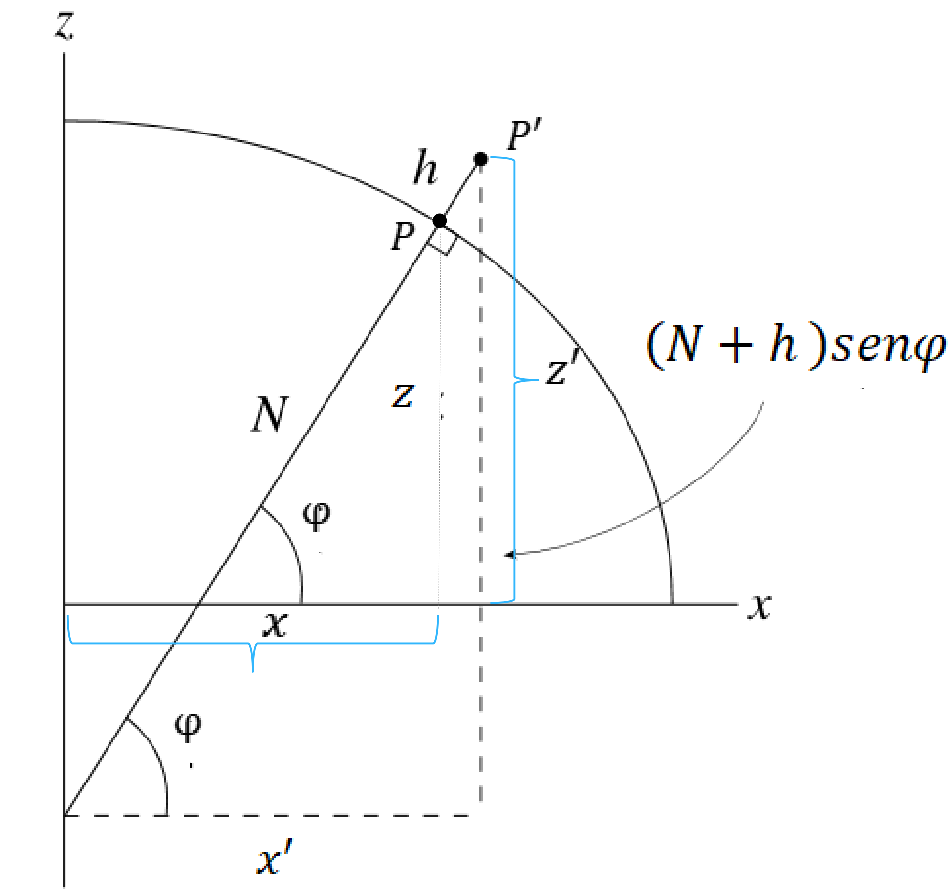
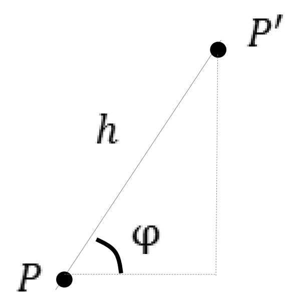
Transformación de Coordenadas[6,7,9]



Fuente: Jekeli, C. (2006)

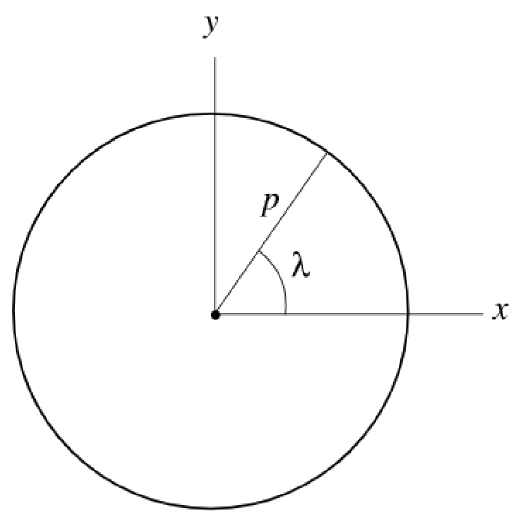
Obtención de ***x*** y ***z*** del punto P

$$\begin{cases} x = \dfrac{a\,cos\varphi}{(1-e^2 sen^2\varphi)^{1/2}} \\[2ex] z = \dfrac{a\,(1-e^2) sen\varphi}{(1-e^2 sen^2\varphi)^{1/2}} \end{cases}$$



$$x' = x + h\,cos\varphi$$

$$z' = z + h\,sen\varphi$$



$$x' = (x + h\,cos\varphi)\,cos\lambda$$

$$y' = (y + h\,cos\varphi)\,sen\lambda$$

(6) Krakiwsky, Edward J., and Donald B. Thomson (1974). *Geodetic position computations*.. Department of Surveying Engineering, University of New Brunswick.
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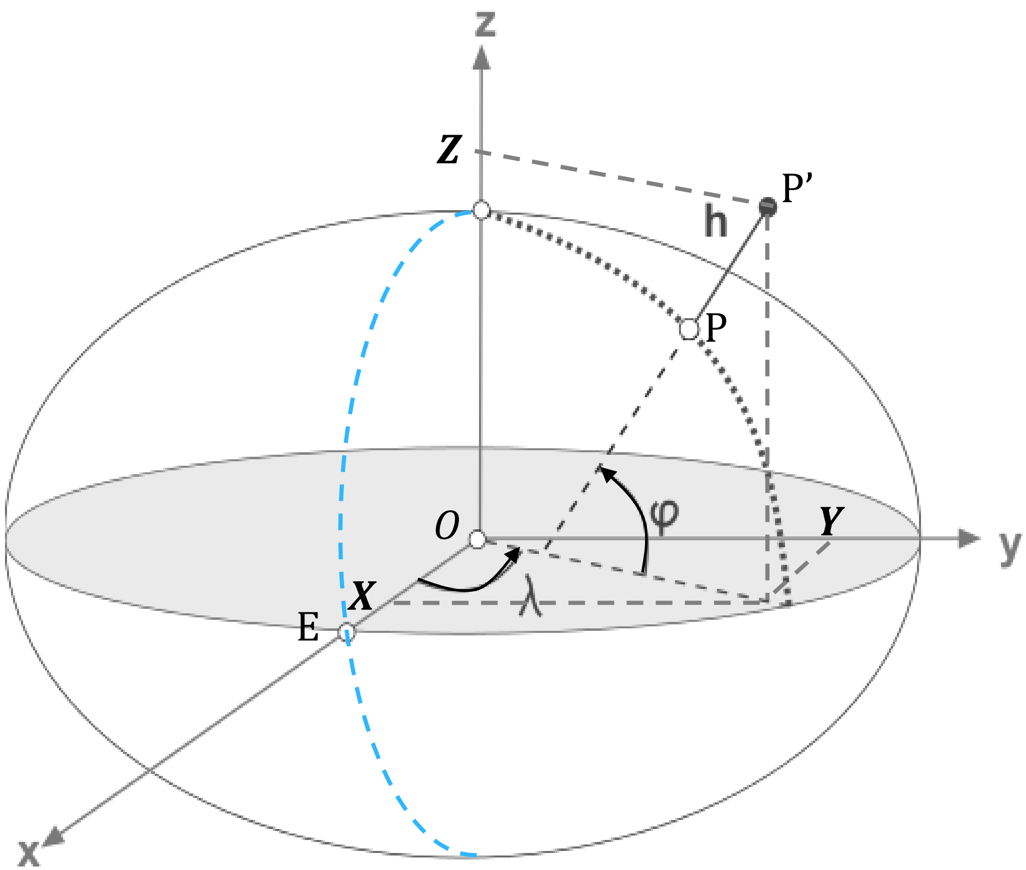
Transformación de Coordenadas(6,7,9)



Fuente: https://gssc.esa.int
ESA: European Space Agency

$$N = \frac{a}{(1 - e^2 sen^2 \varphi)^{1/2}}$$

$$x = \frac{a\, cos\varphi}{(1 - e^2 sen^2 \varphi)^{1/2}}$$

$$z = \frac{a\,(1 - e^2) sen\varphi}{(1 - e^2 sen^2 \varphi)^{1/2}}$$

$$x' = (x + h\, cos\varphi)\, cos\lambda$$

$$y' = (y + h\, cos\varphi)\, sen\lambda$$

$$z' = z + h\, sen\varphi$$

$$X = (N + h) cos\varphi\, cos\lambda$$

$$Y = (N + h)\, cos\varphi\, sen\lambda$$

$$Z = (N(1 - e^2) + h)\, sen\varphi$$

(6) Krakiwsky, Edward J., and Donald B. Thomson (1974). *Geodetic position computations*.. Department of Surveying Engineering, University of New Brunswick.
(7) Rapp, Richard H. (1991). *Geometric geodesy part I*. Department of Geodetic Science and Surveying, Ohio State University.
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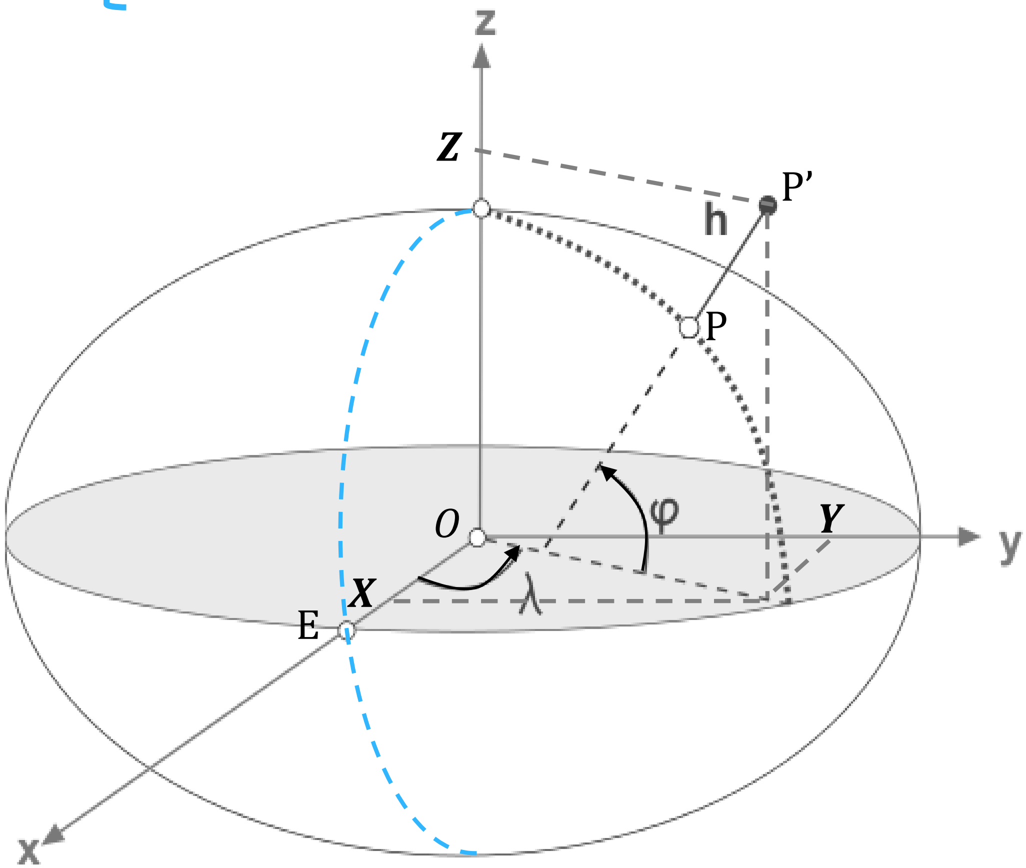
Transformación de Coordenadas[6,7,9]

$$\begin{cases} X = (N + h)\cos\varphi \cos\lambda \\ Y = (N + h)\cos\varphi \, sen\lambda \\ Z = (N(1 - e^2) + h)\, sen\varphi \end{cases}$$

$$\left\{ \frac{Y}{X} = \frac{(N + h)\cos\varphi \, sen\lambda}{(N + h)\cos\varphi \cos\lambda} \rightarrow tg\lambda = \frac{Y}{X} \right.$$



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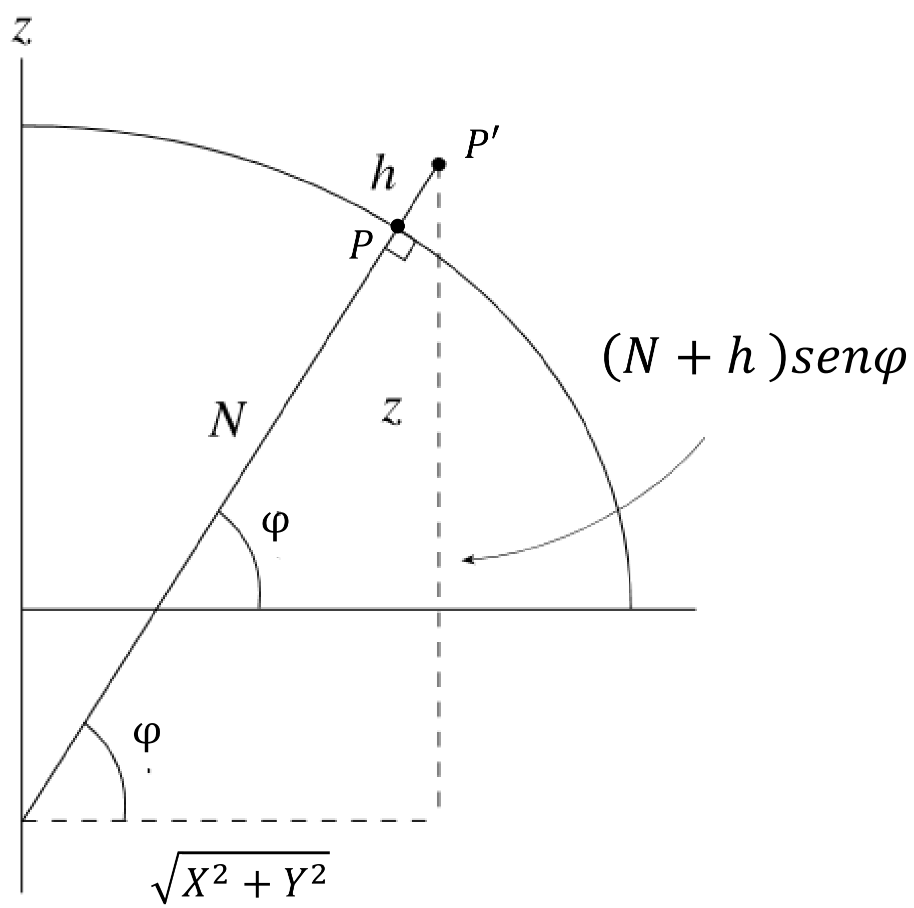
Transformación de Coordenadas[6,7,9] :

$$\begin{cases} X = (N+h)\cos\varphi \, \cos\lambda \\ Y = (N+h)\cos\varphi \, sen\lambda \\ Z = (N(1-e^2)+h)\, sen\varphi \end{cases}$$



Hirvonen & Moritz (1963)

$$tg\varphi = \frac{(N+h)\, sen\,\varphi}{\sqrt{X^2+Y^2}} \quad \rightarrow \quad tg\varphi = \frac{Z + Ne^2 sen\varphi}{\sqrt{X^2+Y^2}}$$

$$\varphi = tg^{-1}\left(\frac{Z}{\sqrt{X^2+Y^2}}\left(1+\frac{Ne^2 sen\varphi}{Z}\right)\right)$$

Si $\rightarrow$ $h = 0 \quad \rightarrow \quad Z = N(1-e^2)\, sen\varphi$

$$\varphi^0 = tg^{-1}\left(\frac{Z}{\sqrt{X^2+Y^2}}\left(1+\frac{e^2}{1-e^2}\right)\right)$$

$$\varphi^1 = tg^{-1}\left(\frac{Z}{\sqrt{X^2+Y^2}}\left(1+\frac{N(\varphi^0)e^2 sen\varphi^0}{Z}\right)\right)$$

$$\varphi^n = tg^{-1}\left(\frac{Z}{\sqrt{X^2+Y^2}}\left(1+\frac{N(\varphi^{n-1})e^2 sen\varphi^{n-1}}{Z}\right)\right),$$

n *iteraciones*

(6) Krakiwsky, Edward J., and Donald B. Thomson (1974). *Geodetic position computations*.. Department of Surveying Engineering, University of New Brunswick.
(7) Rapp, Richard H. (1991). *Geometric geodesy part I*. Department of Geodetic Science and Surveying, Ohio State University.
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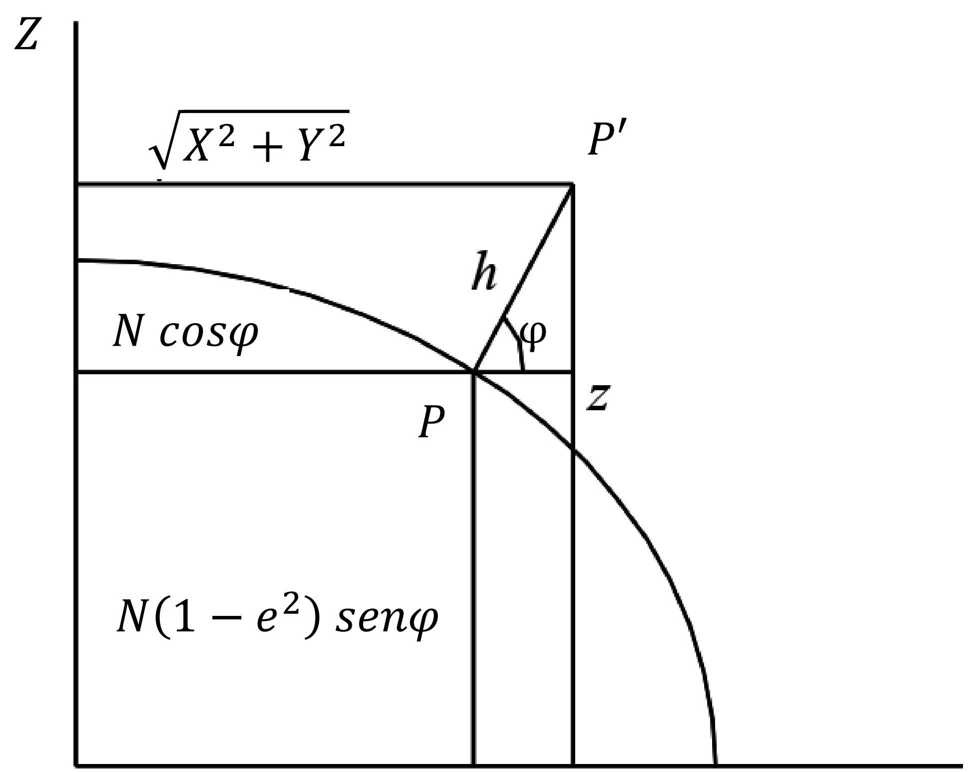
Transformación de Coordenadas[6,7,9] :

$$\begin{cases} X = (N+h)\cos\varphi\,\cos\lambda \\ Y = (N+h)\cos\varphi\,\operatorname{sen}\lambda \\ Z = (N(1-e^2)+h)\operatorname{sen}\varphi \end{cases}$$



Hirvonen & Moritz (1963)

$$\sqrt{X^2+Y^2} = (N+h)\cos\varphi \rightarrow h = \frac{\sqrt{X^2+Y^2}}{\cos\varphi} - N\,, \qquad si\ \varphi \neq 90°$$

$$Z = (N(1-e^2)+h)\operatorname{sen}\varphi \rightarrow h = \frac{Z}{\operatorname{sen}\varphi} - N(1-e^2)\,, \qquad si\ \varphi \neq 0°$$

$$h\cos^2\varphi = (\sqrt{X^2+Y^2} - N\cos\varphi)\cos\varphi$$

$$h\operatorname{sen}^2\varphi = (Z - N(1-e^2)\operatorname{sen}\varphi)\operatorname{sen}\varphi$$

$$h = (\sqrt{X^2+Y^2} - N\cos\varphi)\cos\varphi + (Z - N(1-e^2)\operatorname{sen}\varphi)\operatorname{sen}\varphi$$

(6) Krakiwsky, Edward J., and Donald B. Thomson (1974). *Geodetic position computations*.. Department of Surveying Engineering, University of New Brunswick.
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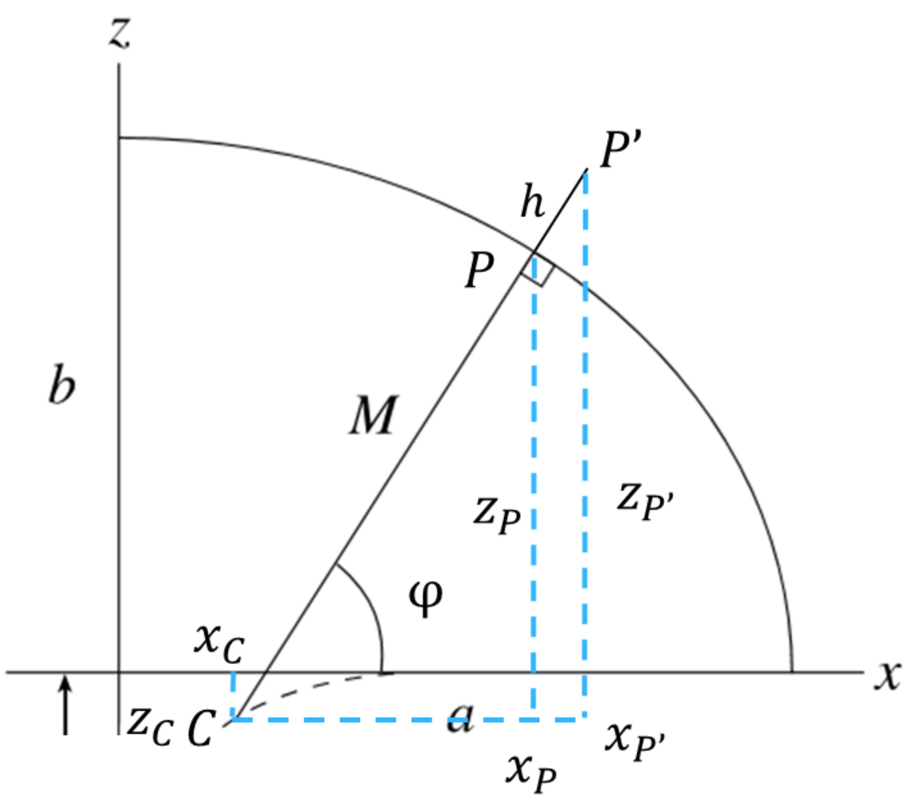
Geodesia

Transformación de Coordenadas: Bowring (1976)

$$\begin{cases} X = (N+h)\cos\varphi\cos\lambda \\ Y = (N+h)\cos\varphi\, sen\lambda \\ Z = (N(1-e^2)+h)\, sen\varphi \end{cases}$$



$$x = \frac{a\cos\varphi}{(1-e^2 sen^2\varphi)^{1/2}}$$

$$z = \frac{a(1-e^2) sen\varphi}{(1-e^2 sen^2\varphi)^{1/2}}$$

$$w^2 = 1 - e^2 sen^2\varphi = \frac{1}{1+e'^2 sen^2\varphi}$$

$$M = \frac{a(1-e^2)}{w^3}$$

$$\cos\beta = \frac{\cos\varphi}{w}$$

$$x_c = x_P - \mathrm{M}\cos\varphi \rightarrow x_c = a\, e^2 \cos^3\beta$$

$$z_c = z_P - \mathrm{M}\, sen\varphi \rightarrow z_c = -b\, e'^2\, sen^3\beta$$

$$tg\varphi = \frac{z_{P'} - z_C}{x_{P'} - x_C} = \frac{z_{P'} + b\, e'^2\, sen^3\beta}{x_{P'} - a\, e^2 \cos^3\beta}$$

$$x_{P'} = \sqrt{X^2+Y^2}$$

$$tg\varphi = \frac{z_{P'} + b\, e'^2\, sen^3\beta}{\sqrt{X^2+Y^2} - a\, e^2\cos^3\beta}$$

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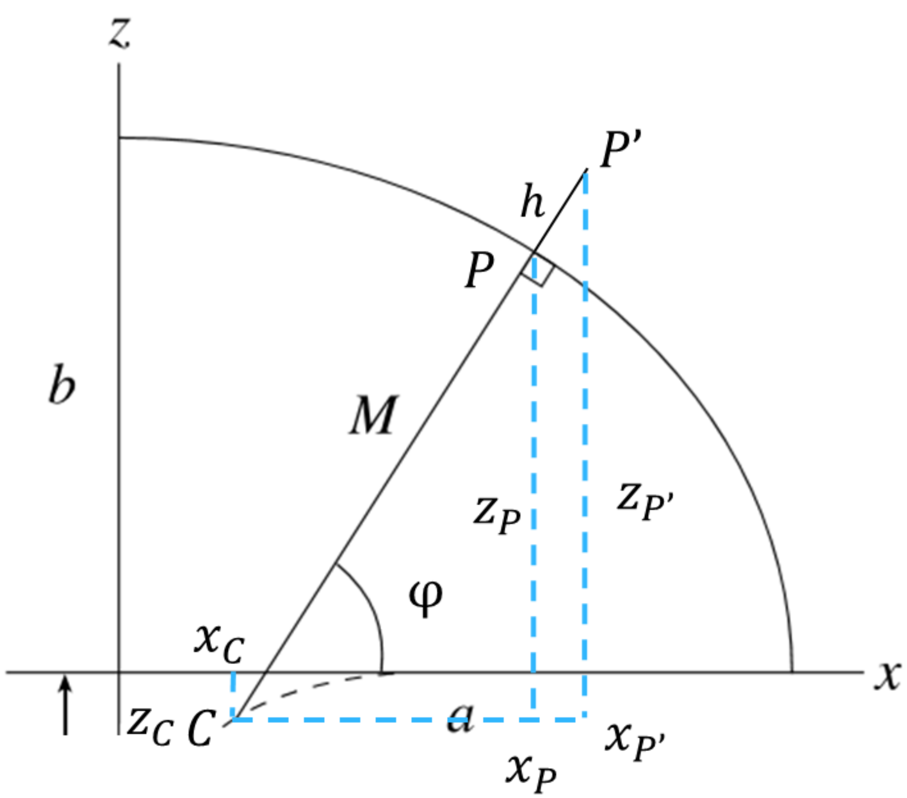

$$x = \frac{a\cos\varphi}{(1 - e^2 sen^2\varphi)^{1/2}}$$

$$z = \frac{a\,(1 - e^2) sen\varphi}{(1 - e^2 sen^2\varphi)^{1/2}}$$

$$w^2 = 1 - e^2 sen^2\varphi = \frac{1}{1 + e'^2 sen^2\varphi}$$

$$M = \frac{a(1 - e^2)}{w^3}$$

$$\cos\beta = \frac{\cos\varphi}{w}$$

$$z = b\, sen\beta$$
$$x = a\cos\beta$$

$$\rightarrow \quad tg\beta^0 = \frac{a}{b}\frac{z_{P'}}{\sqrt{X^2+Y^2}}$$

$$tg\varphi^0 = \frac{z_{P'} + b\, e'^2\, sen^3\beta^0}{\sqrt{X^2+Y^2} - a\, e^2\, \cos^3\beta^0}$$

$$tg\beta^1 = (1 - f)\, tg\varphi^0$$

$$tg\varphi^n = \frac{z_{P'} + b\, e'^2\, sen^3\beta^{n-1}}{\sqrt{X^2+Y^2} - a\, e^2\, \cos^3\beta^{n-1}}$$

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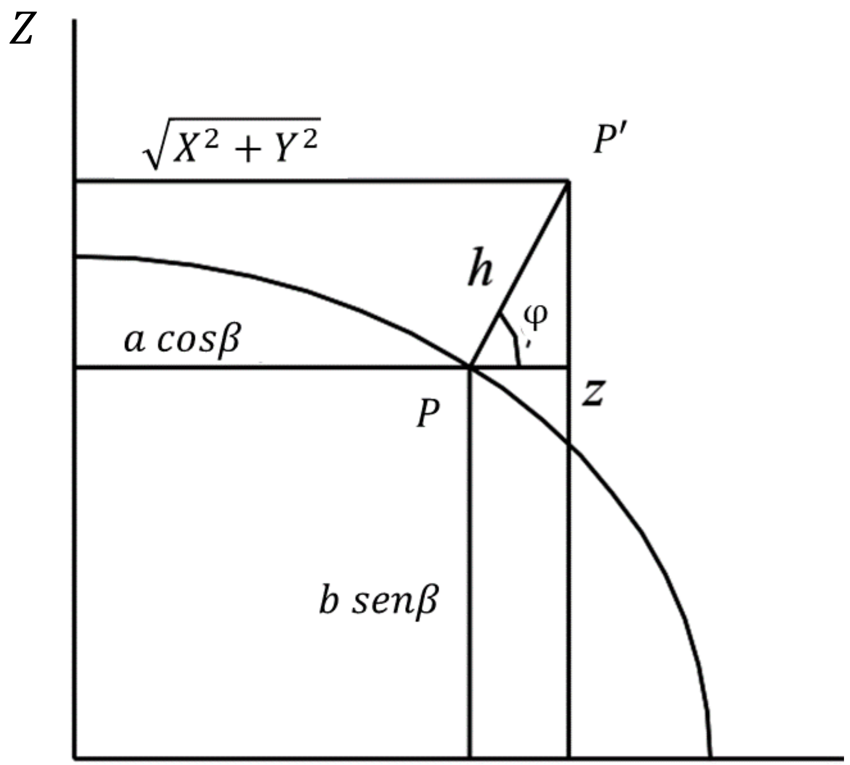

$$h^2 = (\sqrt{X^2 + Y^2} - a\cos\beta)^2 + (z - b\,sen\beta)^2$$ (Bartelme & Meissl, 1975)

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