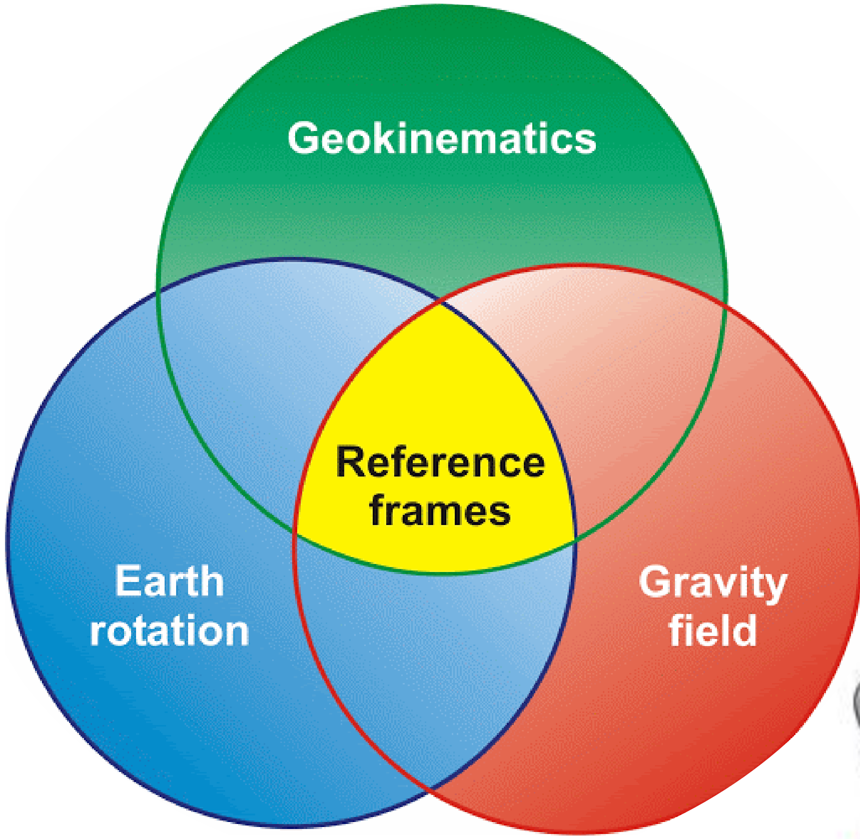
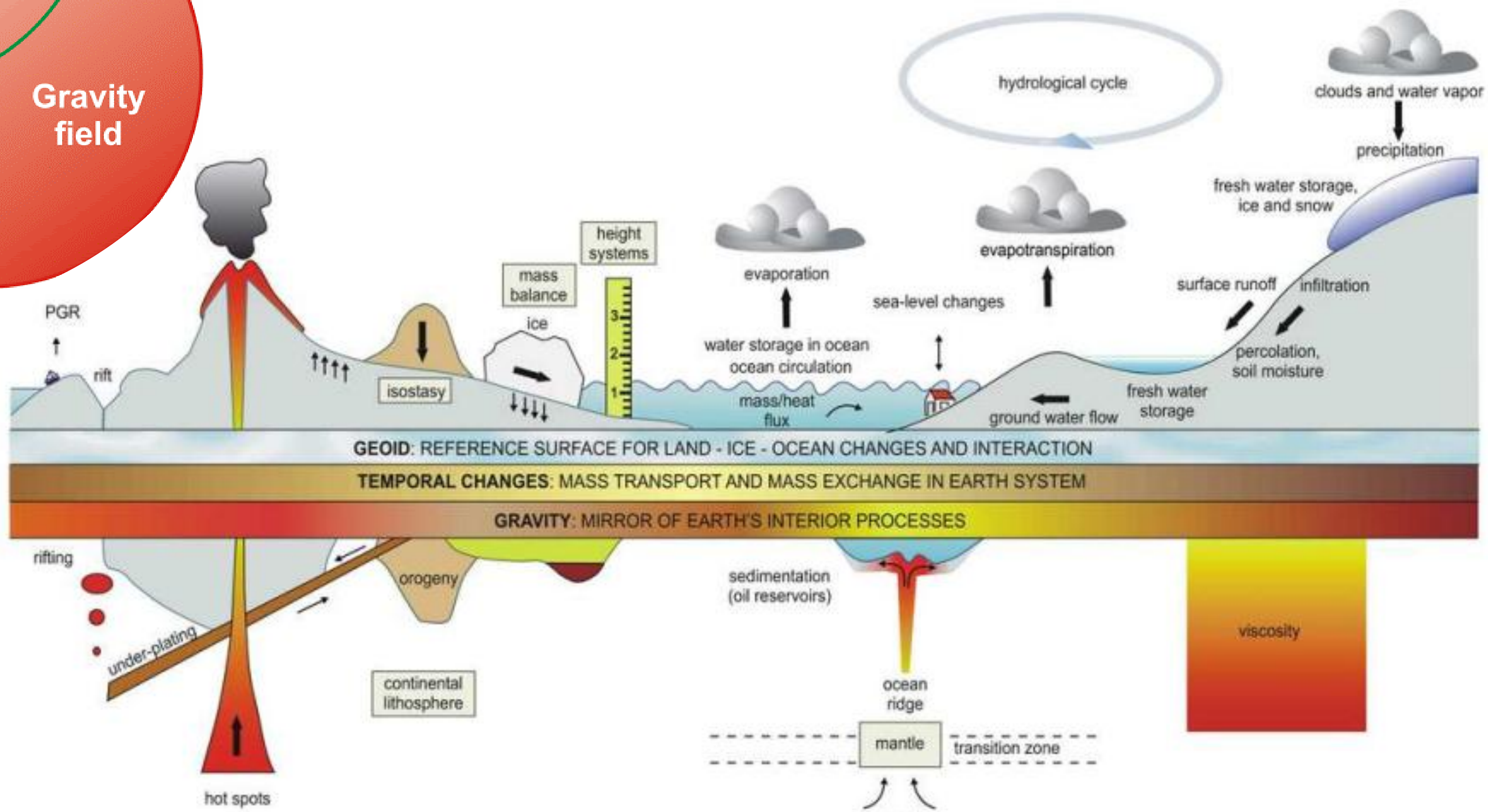


GEODESIA

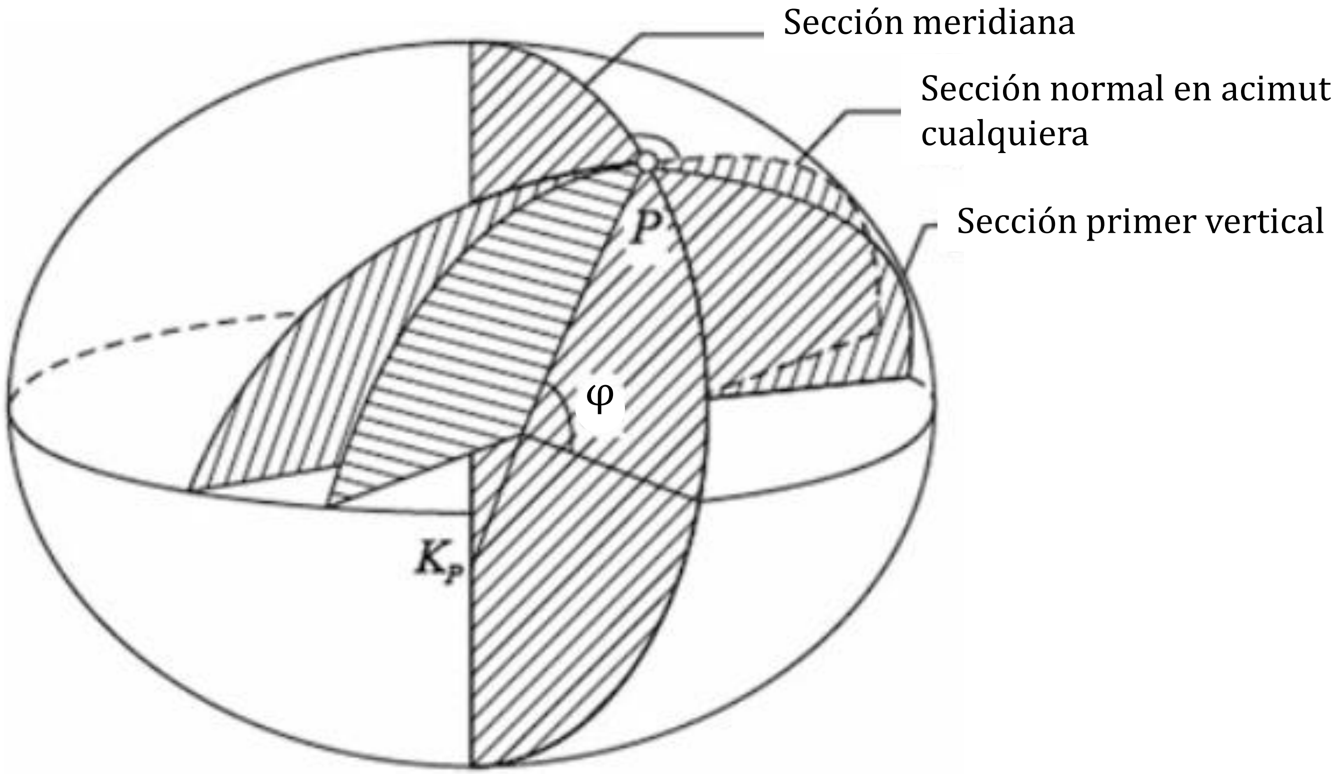


Plag, H. P., Rothacher, M., Pearlman, M., Neilan, R., & Ma, C. (2009). The global geodetic observing system. In *Advances in Geosciences: Volume 13: Solid Earth (SE)* (pp. 105-127).

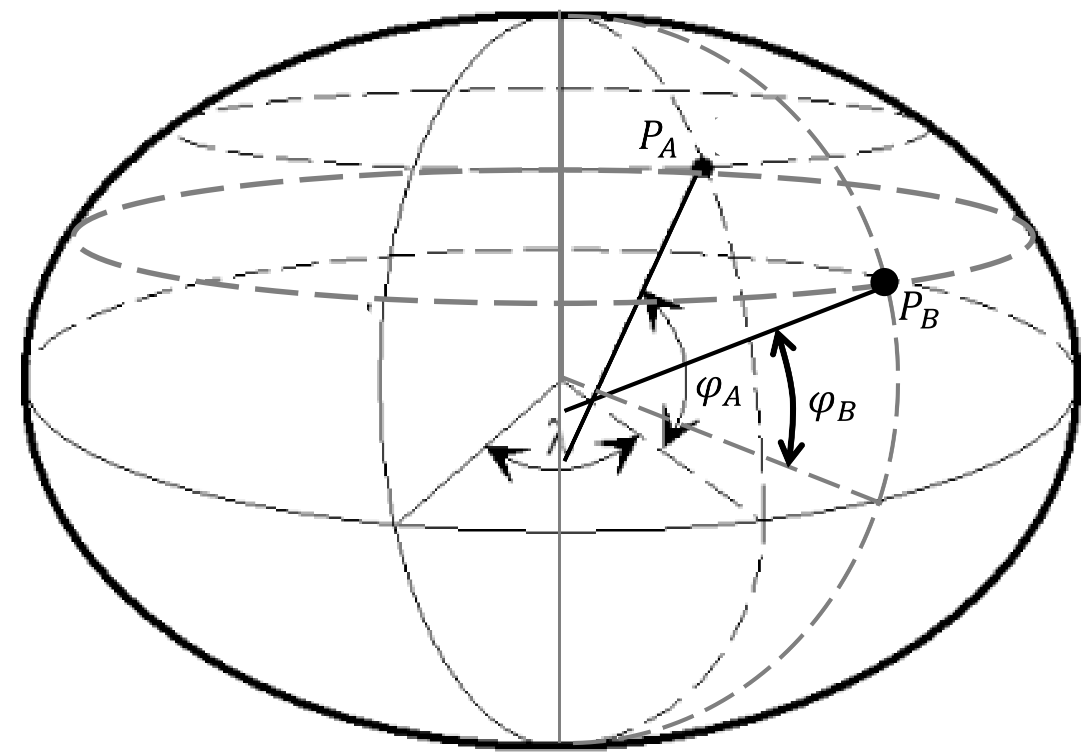
Geodesia

Secciones en el elipsoide[7,9,10]**:**

Sección Normal (definición matemática): curva formada por la intersección de un plano que contiene a la normal a la superficie del elipsoide en un punto dado.



Fuente: Lu Z., Qu Y., Qiao S. (2014)



(7) Rapp, Richard H. (1991). *Geometric geodesy part I*. Department of Geodetic Science and Surveying, Ohio State University.
(9) Jekeli, C. (2006). *Geometric reference systems in geodesy*. Division of Geodetic Science, School of Earth Sciences. Ohio State University.
(10) Jordan, W. (1962). *Handbook of Geodesy* (Vol. 3). Corps of Engineers, United States Army, Army Map Service

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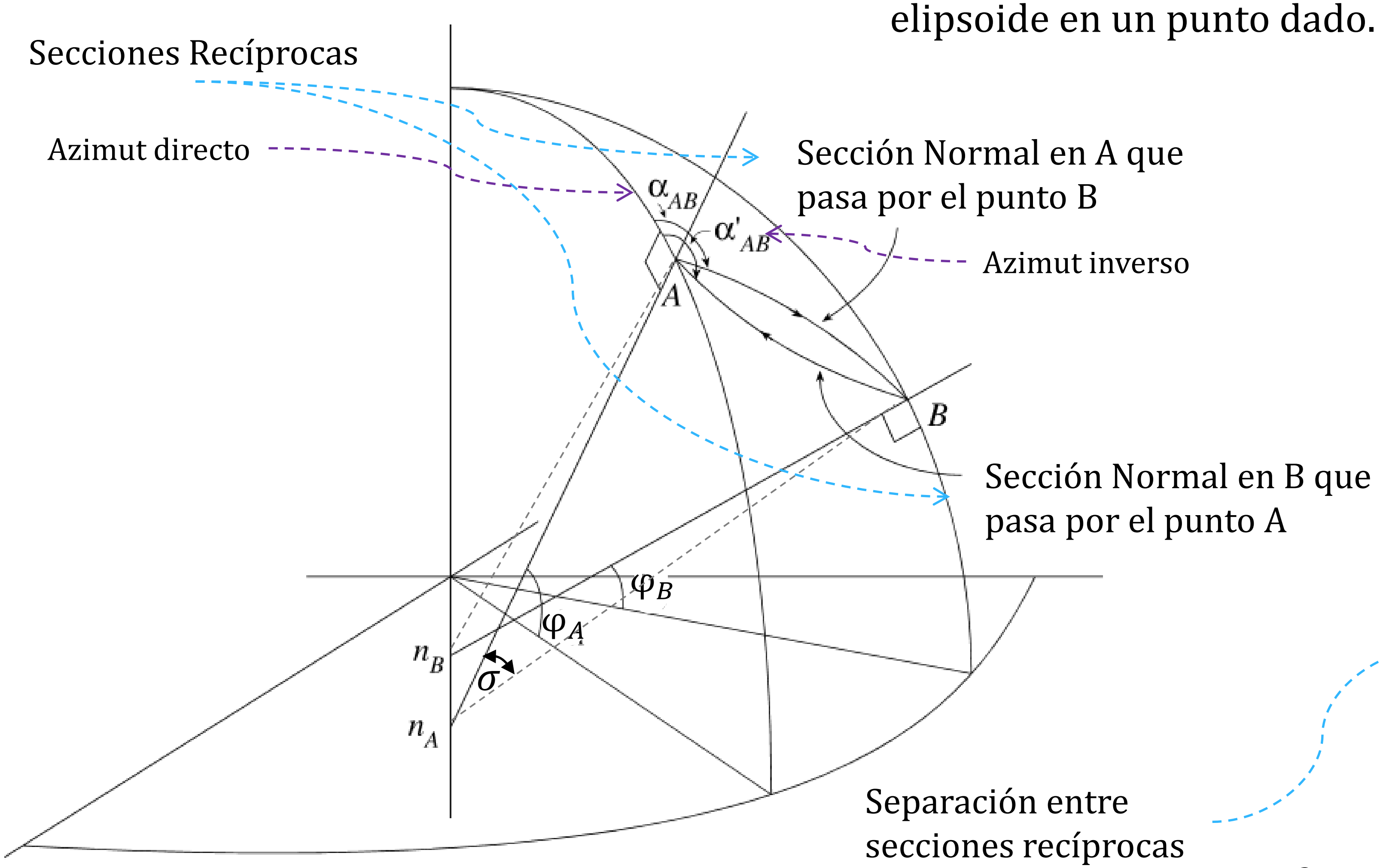
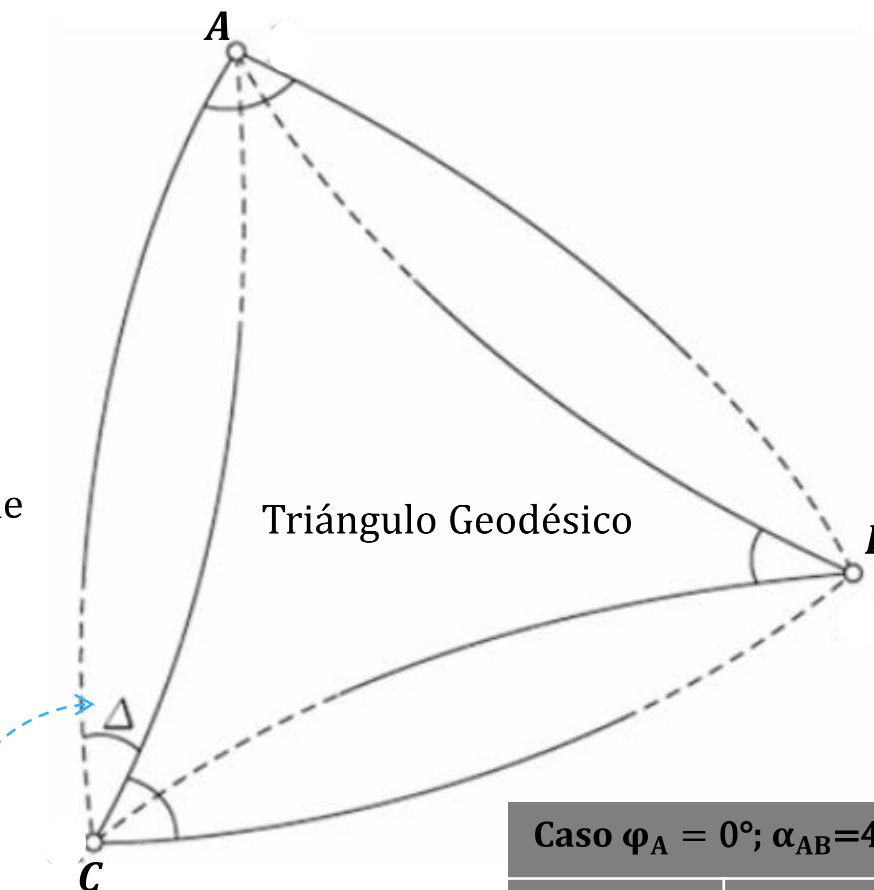




Fuente: Jekeli, C. (2006)

$$\Delta = \alpha_{AB} - \alpha'_{AB} = \frac{e^2}{4}\left(\frac{S_{AB}}{N_A}\right)^2 sen2\alpha_{AB}\, cos^2\varphi_m$$

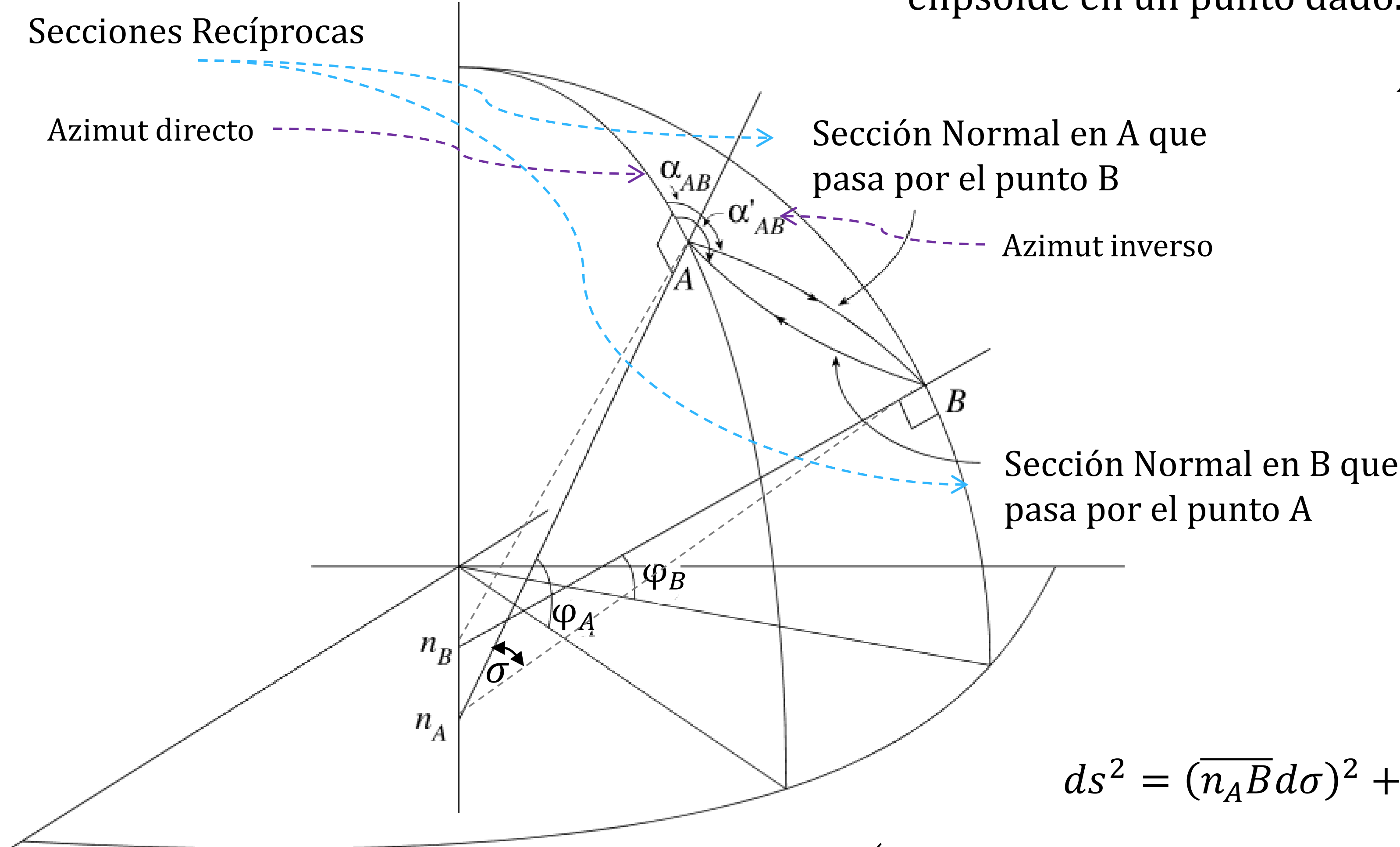
Caso $\varphi_A = 0°$; $\alpha_{AB}=45°$	
S (km)	**Δ (")**
200	0.399
100	0.085
50	0.021

(7) Rapp, Richard H. (1991). *Geometric geodesy part I*. Department of Geodetic Science and Surveying, Ohio State University.
(9) Jekeli, C. (2006). *Geometric reference systems in geodesy*. Division of Geodetic Science, School of Earth Sciences. Ohio State University.
(10) Jordan, W. (1962). *Handbook of Geodesy* (Vol. 3). Corps of Engineers, United States Army, Army Map Service

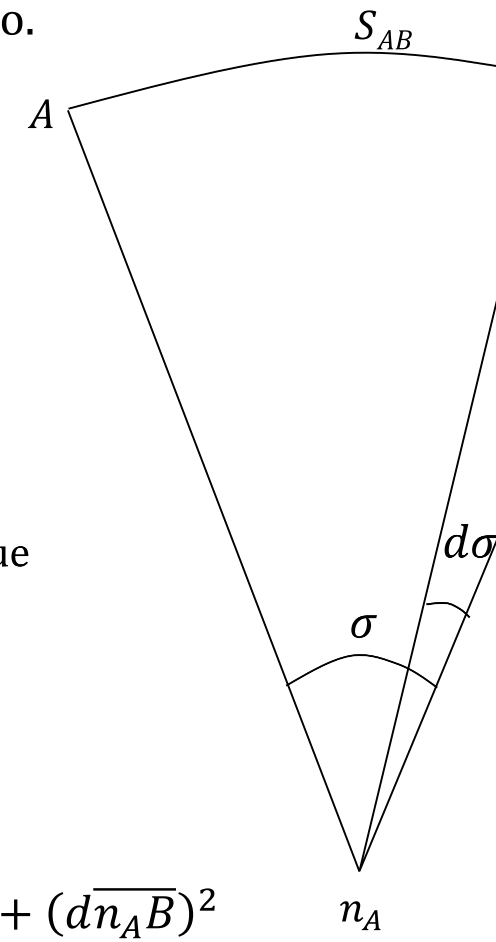
Geodesia

Secciones en el elipsoide[7,9,10]:

Sección Normal (definición matemática): curva formada por la intersección de un plano que contiene a la normal a la superficie del elipsoide en un punto dado.



Fuente: Jekeli, C. (2006)



$$\eta^2 = e'^2 cos^2\varphi$$

$$t = tg\varphi$$

$$ds^2 = (\overline{n_A B} d\sigma)^2 + (d\overline{n_A B})^2$$

$$S_{AB} = N_A \sigma \left(1 + \frac{1}{6}\sigma^2 \eta_A{}^2 \cos^2\alpha_{AB}(\eta_A{}^2 \cos^2\alpha_{AB} - 1) + \frac{1}{8} t_A \eta_A{}^2 \cos\alpha_{AB}(1 - 3\,\eta_A{}^2 \cos^2\alpha_{AB})\sigma^3\right)$$

(7) Rapp, Richard H. (1991). *Geometric geodesy part I*. Department of Geodetic Science and Surveying, Ohio State University.
(9) Jekeli, C. (2006). *Geometric reference systems in geodesy*. Division of Geodetic Science, School of Earth Sciences. Ohio State University.
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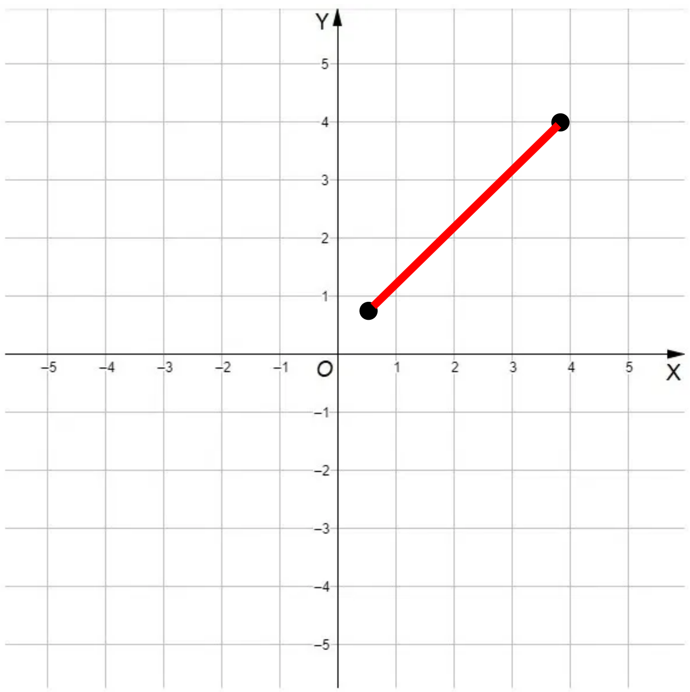
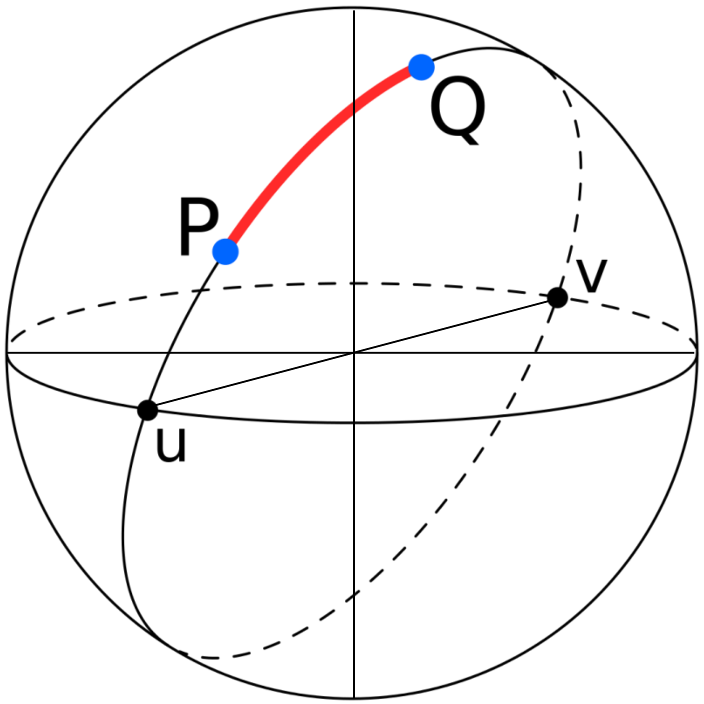
Geodesia

Geodésicas[7,9,10]: Longitud más corta entre dos puntos sobre una superficie.

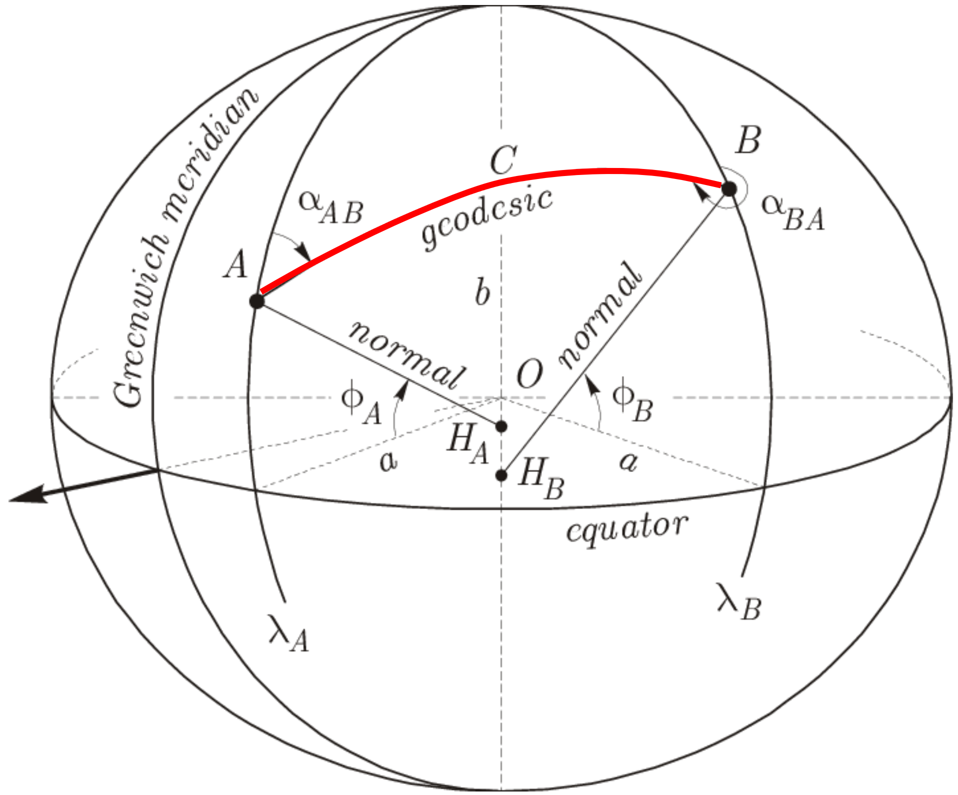
Geodésica en el plano



Geodésica en la esfera



Geodésica en el elipsoide



Fuente: Deakin, R. E., (2005).
Traverse computation: Ellipsoid versus the UTM projection.

(7) Rapp, Richard H. (1991). *Geometric geodesy part I*. Department of Geodetic Science and Surveying, Ohio State University.
(9) Jekeli, C. (2006). *Geometric reference systems in geodesy*. Division of Geodetic Science, School of Earth Sciences. Ohio State University.
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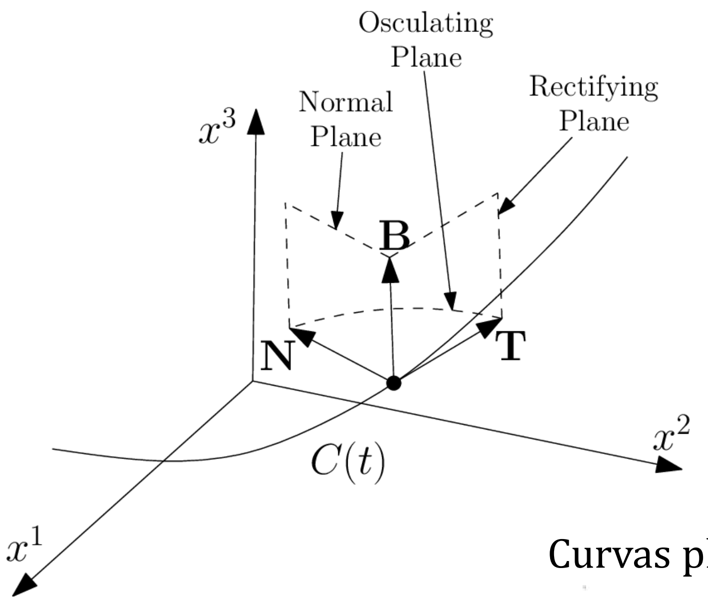
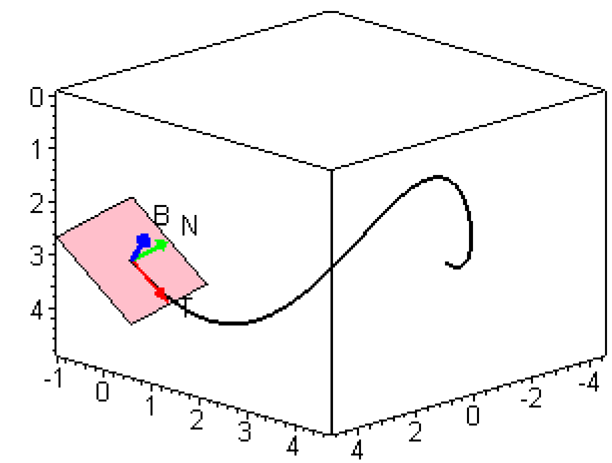
Geodesia

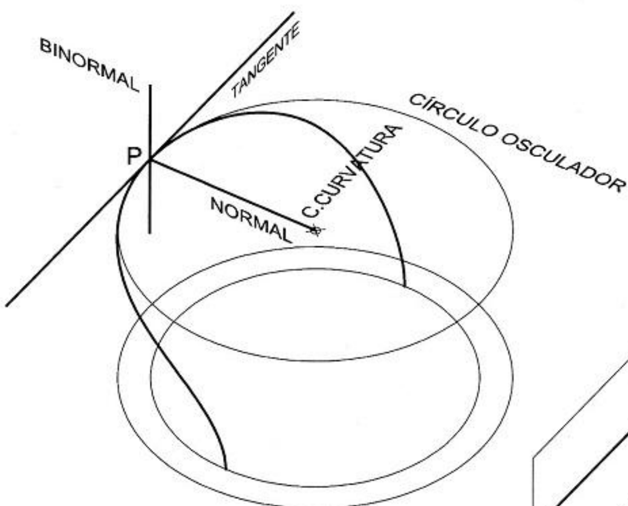
<u>Geodésicas[7,9,10]</u>: toda curva sobre una superficie cuyo plano osculador contiene a la normal a la superficie en cualquier punto.

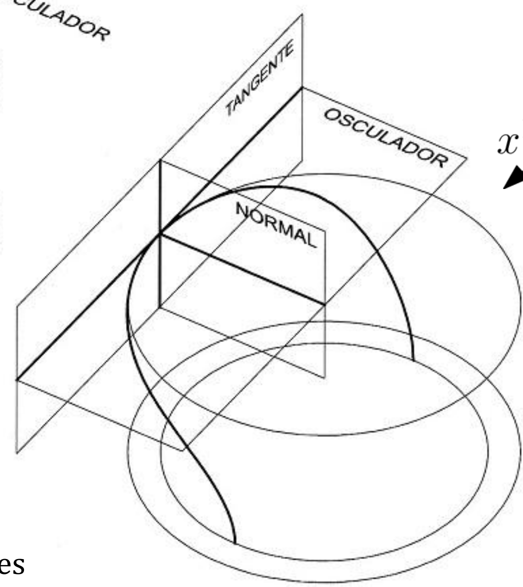




Fuente: http://faculty.etsu.edu

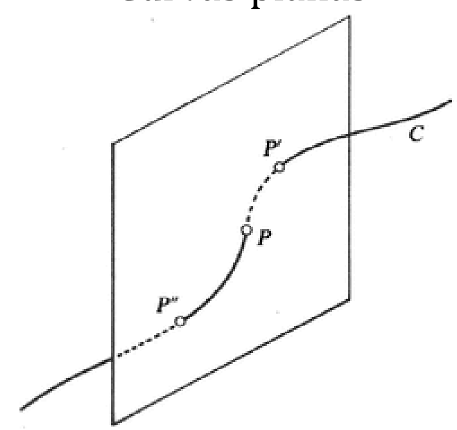
Triedo de Frenet



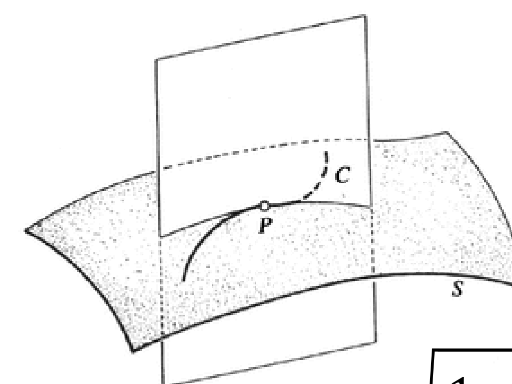


Fuente: http://www.edificacion.upm.es

Curvas planas



Doble curvatura



$$\sqrt{\frac{1}{\chi^2}+\frac{1}{\tau^2}}$$

(7) Rapp, Richard H. (1991). *Geometric geodesy part I*. Department of Geodetic Science and Surveying, Ohio State University.
(9) Jekeli, C. (2006). *Geometric reference systems in geodesy*. Division of Geodetic Science, School of Earth Sciences. Ohio State University.
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Geodesia

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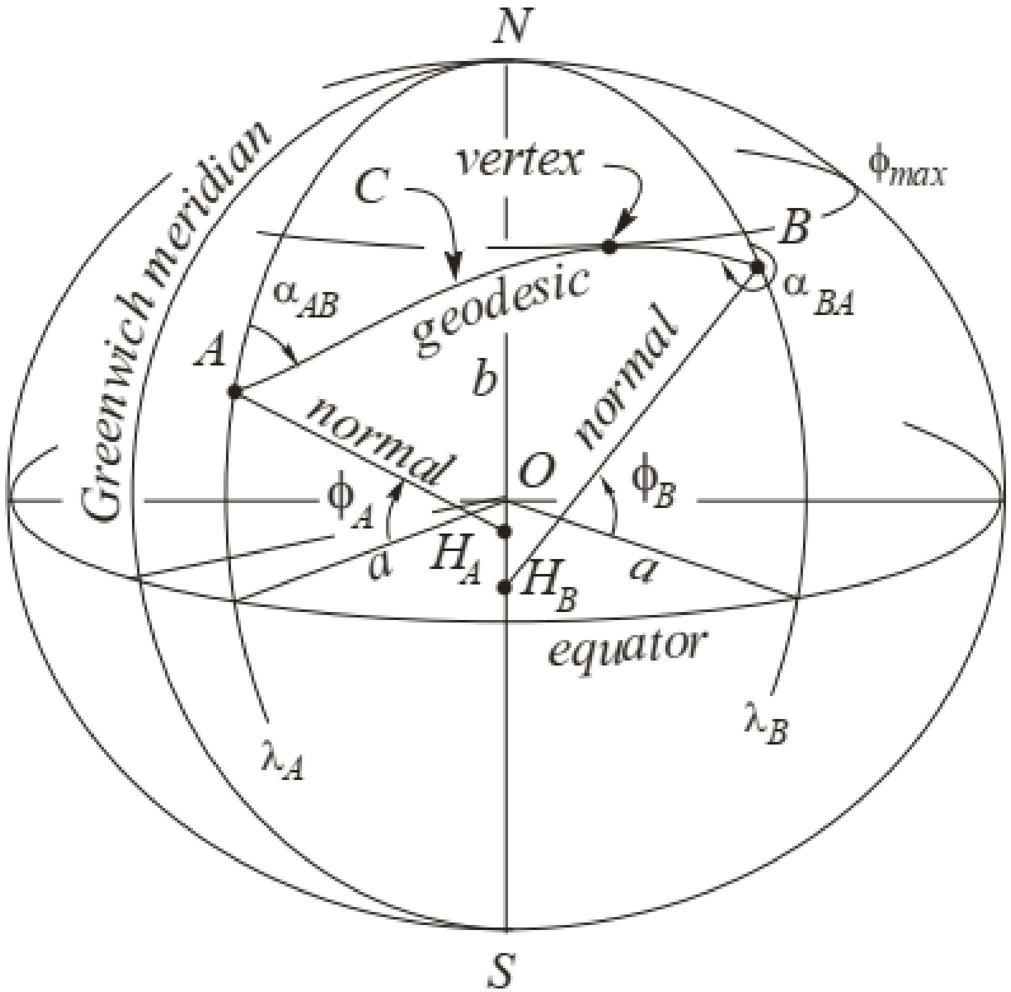
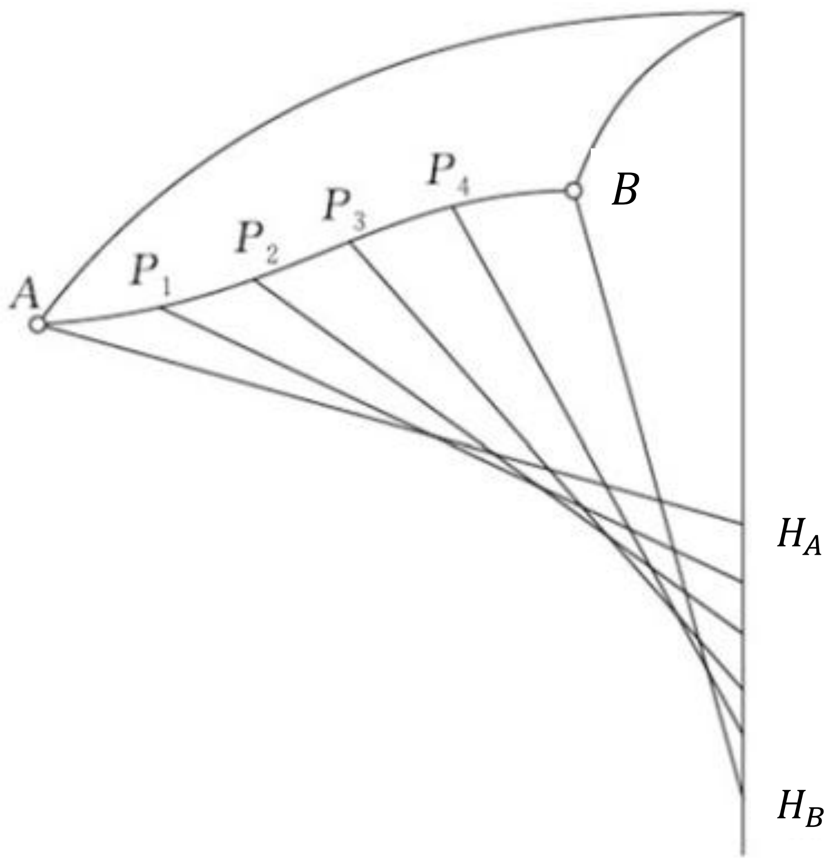


Fuente: Deakin, R. E., & Hunter, M. N. (2009).



Fuente: Lu Z., Qu Y., Qiao S. (2014)

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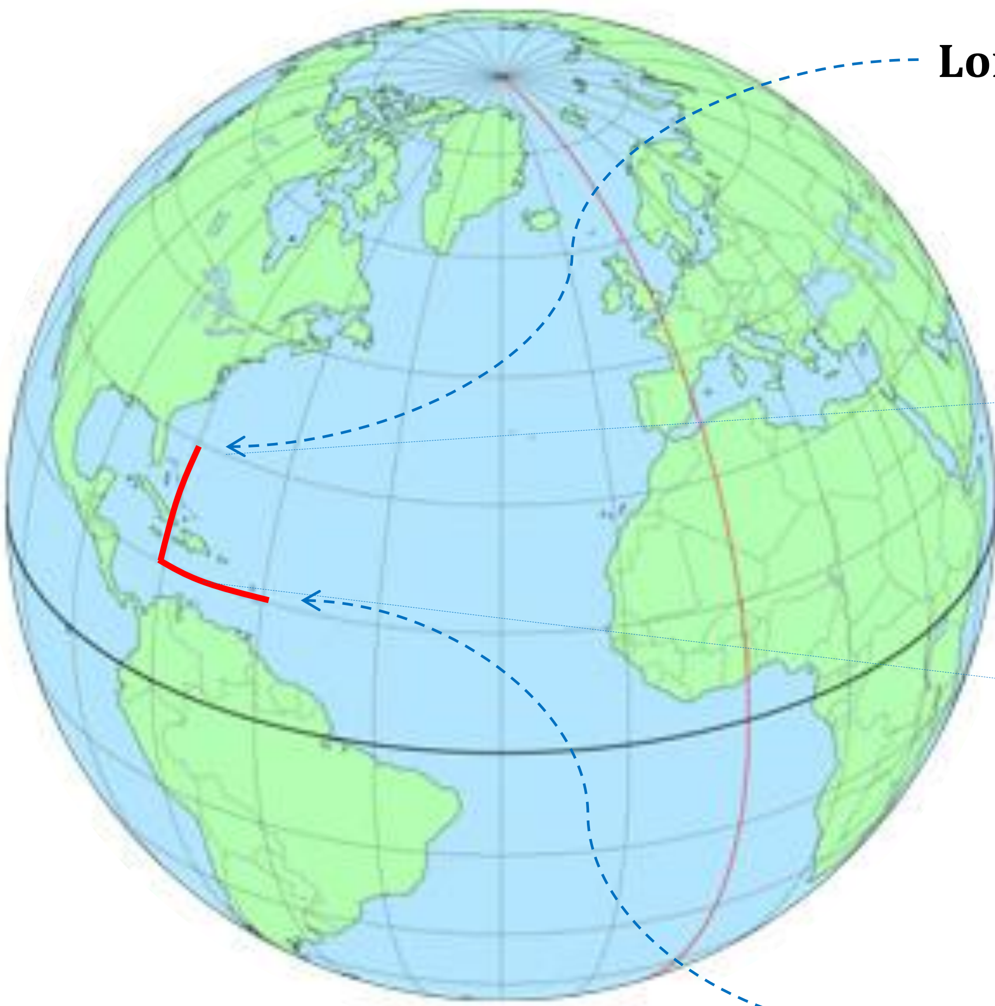
(9) Jekeli, C. (2006). *Geometric reference systems in geodesy*. Division of Geodetic Science, School of Earth Sciences. Ohio State University.
(10) Jordan, W. (1962). *Handbook of Geodesy* (Vol. 3). Corps of Engineers, United States Army, Army Map Service

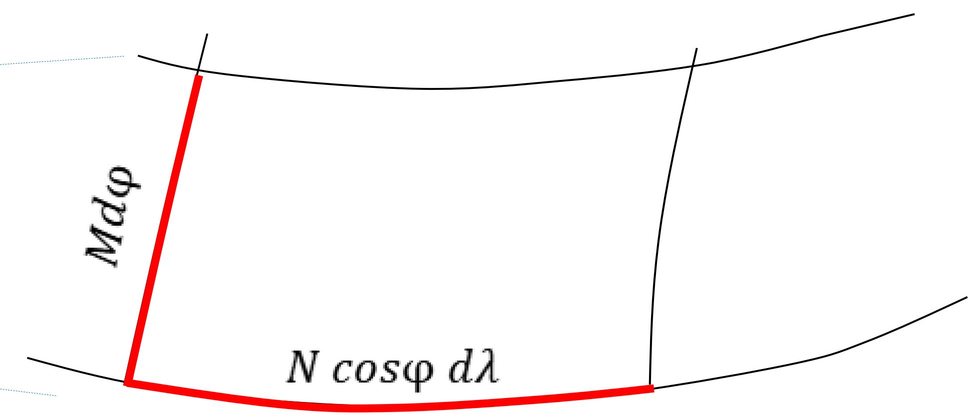
Geodesia

Longitudes de arcos[7,9,10]:



Longitud de elemento de Arco de Meridiano

$$\chi = \left|\frac{d\alpha}{ds}\right| \rightarrow \frac{1}{M} = \left|\frac{d\varphi}{ds}\right| \Longrightarrow ds_{meridiana} = M d\varphi$$



$$\chi = \left|\frac{d\alpha}{ds}\right| \rightarrow \frac{1}{p} = \left|\frac{d\lambda}{ds}\right| \Longrightarrow ds_{paralelo} = N\, cos\varphi\, d\lambda$$

Longitud de elemento de Arco de Paralelo

Fuente: www.thegreenwichmeridian.org

(7) Rapp, Richard H. (1991). *Geometric geodesy part I*. Department of Geodetic Science and Surveying, Ohio State University.
(9) Jekeli, C. (2006). *Geometric reference systems in geodesy*. Division of Geodetic Science, School of Earth Sciences. Ohio State University.
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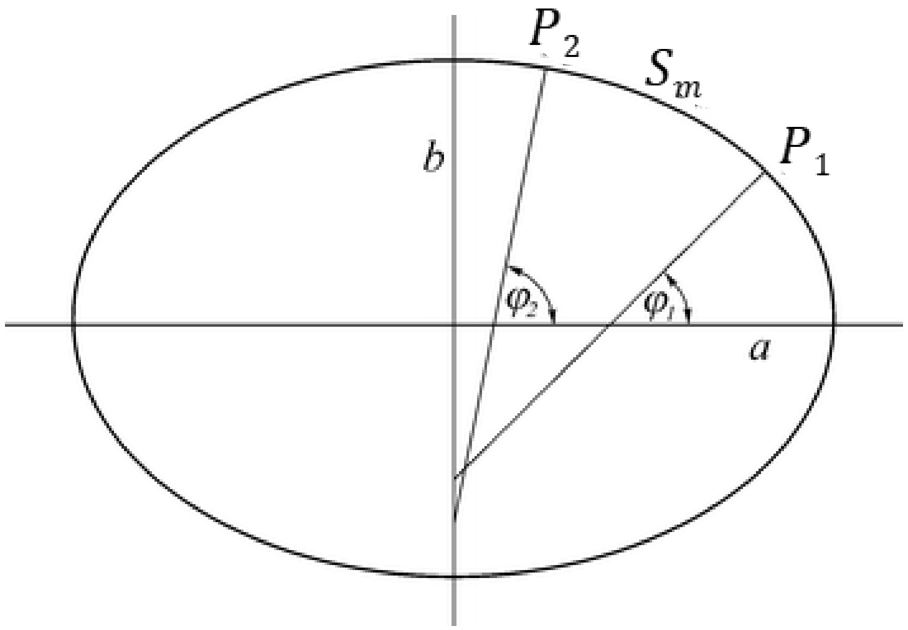
Geodesia

Longitudes de arcos[7,9,10]:

$$\chi = \left|\frac{d\alpha}{ds}\right| \rightarrow \frac{1}{M} = \left|\frac{d\varphi}{ds}\right| \Longrightarrow ds_{meridiana} = M d\varphi$$



Fuente: Lapaine M. (2017)

$$S_m = \int_{\varphi_1}^{\varphi_2} M d\varphi = \int_{\varphi_1}^{\varphi_2} \frac{a(1-e^2)}{(1-e^2 sen^2\varphi)^{3/2}} d\varphi$$

$$S_m = a(1-e^2)\int_{\varphi_1}^{\varphi_2} (1-e^2 sen^2\varphi)^{-3/2} d\varphi$$

Integral elíptica

Series de Taylor (Maclaurin)

$$f(x) = f(x_0) + (x-x_0)f'(x_0) + \frac{(x-x_0)^2}{2!} f''(x_0) + \dots$$

$$x_0 = 0\,; f(x) = f(0) + x\, f'(0) + \frac{x^2}{2!} f''(0) + \frac{x^3}{3!} f'''(0) +$$

$$\frac{1}{(1-e^2 sen^2\varphi)^{3/2}} = 1 + \frac{3}{2}e^2 sen^2\varphi + \frac{15}{8}e^4 sen^4\varphi + \frac{35}{16}e^6 sen^6\varphi + \frac{315}{128}e^8 sen^8\varphi + \frac{693}{256}e^{10} sen^{10}\varphi + \dots$$

$$sen^2\varphi = \frac{1}{2} - \frac{1}{2}cos2\varphi\,;\ sen^4\varphi = \frac{3}{8} - \frac{1}{2}cos2\varphi + \frac{1}{8}cos4\varphi; \dots$$

$$\frac{1}{(1-e^2 sen^2\varphi)^{3/2}} = A - B\,cos2\varphi + C\,cos4\varphi - D\,cos6\varphi + E\,cos8\varphi - F\,cos10\varphi + \dots$$

$$\int cos2\varphi\, d\varphi = \frac{1}{2}sen2\varphi$$

(7) Rapp, Richard H. (1991). *Geometric geodesy part I*. Department of Geodetic Science and Surveying, Ohio State University.
(9) Jekeli, C. (2006). *Geometric reference systems in geodesy*. Division of Geodetic Science, School of Earth Sciences. Ohio State University.
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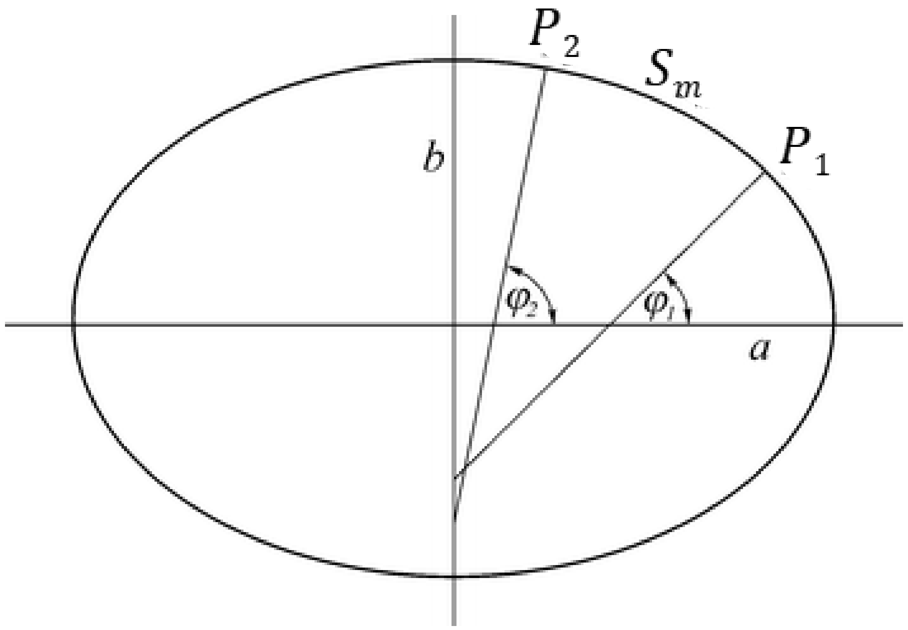
Geodesia

Longitudes de arcos[7,9,10]:



Fuente: Lapaine M. (2017)

$$S_m = a(1-e^2)\Big[A(\varphi_2-\varphi_1) - \tfrac{B}{2}(sen2\varphi_2 - sen2\varphi_1) + \tfrac{C}{4}(sen4\varphi_2 - sen4\varphi_1) - \tfrac{D}{6}(sen6\varphi_2 - sen6\varphi_1) + \tfrac{E}{8}(sen8\varphi_2 - sen8\varphi_1) - \tfrac{F}{10}(sen10\varphi_2 - sen10\varphi_1)\Big]$$

$$A = 1 + \frac{3}{4}e^2 + \frac{45}{64}e^4 + \frac{175}{256}e^6 + \frac{11025}{16384}e^8 + \frac{43659}{65536}e^{10} + \dots$$

$$B = \frac{3}{4}e^2 + \frac{15}{16}e^4 + \frac{525}{512}e^6 + \frac{2205}{2048}e^8 + \frac{72765}{65536}e^{10} + \dots$$

$$C = \frac{15}{64}e^4 + \frac{105}{256}e^6 + \frac{2205}{4096}e^8 + \frac{10395}{16384}e^{10} + \dots$$

$$D = \frac{35}{512}e^6 + \frac{315}{2048}e^8 + \frac{31185}{131072}e^{10} + \dots$$

$$E = \frac{315}{16384}e^8 + \frac{3465}{65536}e^{10} + \dots$$

$$F = \frac{693}{131072}e^{10} + \dots$$

Fuente: Jordan, W. (1962).

Si $\varphi_1 = 0° \rightarrow$ *Longitud de arco desde el ecuador al polo*

(7) Rapp, Richard H. (1991). *Geometric geodesy part I*. Department of Geodetic Science and Surveying, Ohio State University.
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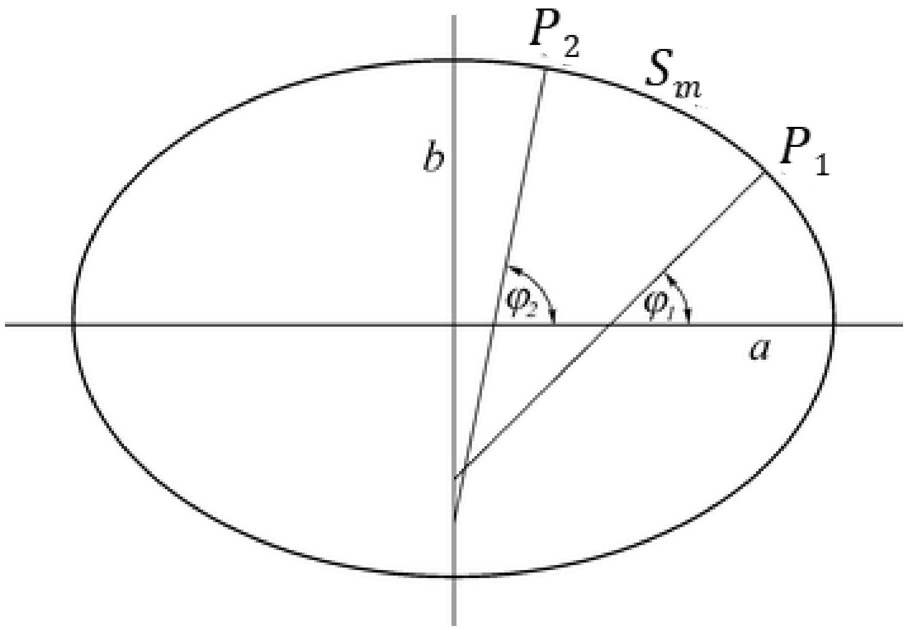
(10) Jordan, W. (1962). *Handbook of Geodesy* (Vol. 3). Corps of Engineers, United States Army, Army Map Service

Geodesia

Longitudes de arcos[7,9,10]:



Fuente: Lapaine M. (2017)

Si $\varphi_1 = 0°$ $\rightarrow$ *Longitud de arco desde el ecuador al polo*

Helmert

$$S_m = \frac{a}{1+n}[a_0\varphi_2 - a_2 sen2\varphi_2 + a_4 sen4\varphi_2 - a_6 sen6\varphi_2 + a_8 sen8\varphi_2]$$

$$a_0 = 1 + \frac{n^2}{4} + \frac{n^4}{64} \qquad n = \frac{f}{2-f} = \frac{1-\sqrt{1-e^2}}{1+\sqrt{1-e^2}}$$

$$a_2 = \frac{3}{2}\left(n - \frac{n^3}{8}\right)$$

$$a_4 = \frac{15}{16}\left(n^2 - \frac{n^4}{4}\right)$$

$$a_6 = \frac{35}{45}n^3$$

$$a_8 = \frac{315}{512}n^4$$

(7) Rapp, Richard H. (1991). *Geometric geodesy part I*. Department of Geodetic Science and Surveying, Ohio State University.
(9) Jekeli, C. (2006). *Geometric reference systems in geodesy*. Division of Geodetic Science, School of Earth Sciences. Ohio State University.
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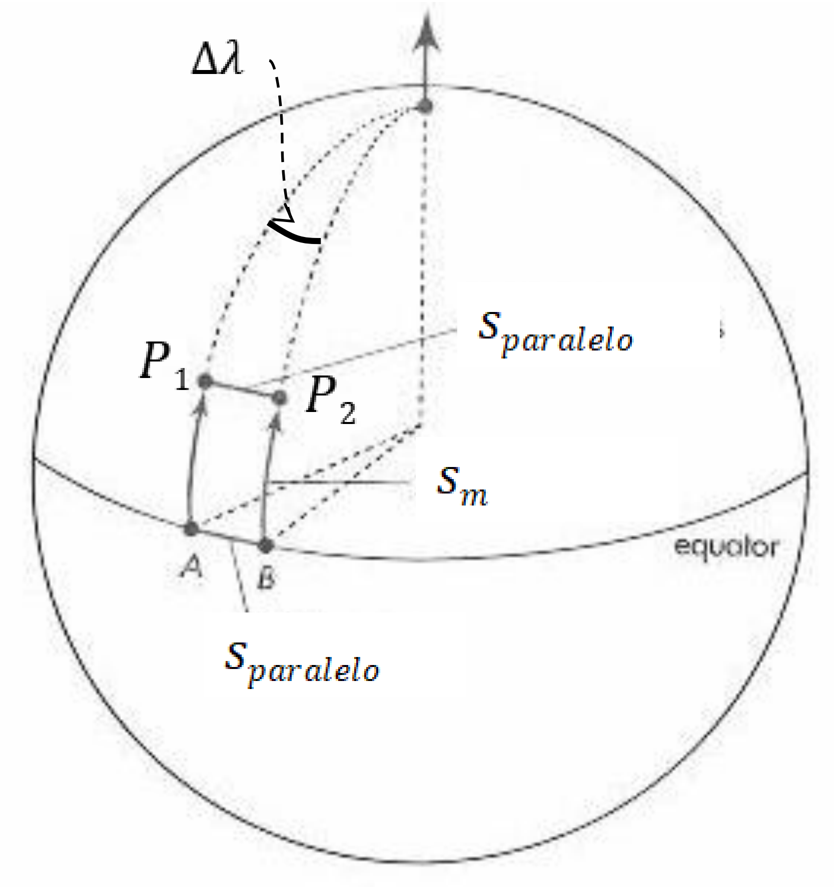
Longitudes de arcos[7,9,10]:



$$\chi = \left|\frac{d\alpha}{ds}\right| \rightarrow \frac{1}{p} = \left|\frac{d\lambda}{ds}\right| \Longrightarrow ds_{paralelo} = N\, cos\varphi\, d\lambda$$

$$s_{paralelo} = N\, cos\varphi\, \Delta\lambda \quad , \quad \Delta\lambda = \lambda_2 - \lambda_1$$

(7) Rapp, Richard H. (1991). *Geometric geodesy part I*. Department of Geodetic Science and Surveying, Ohio State University.
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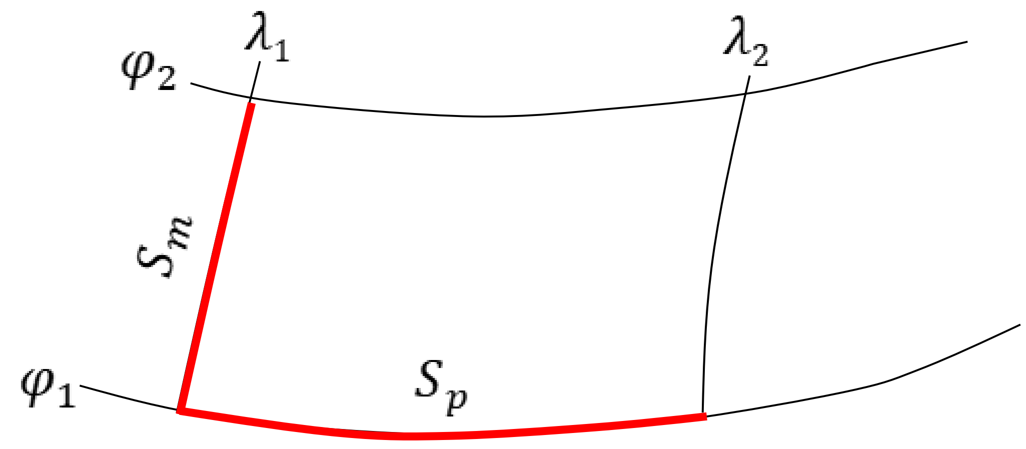
Geodesia

Longitudes de arcos[7,9,10]:

Longitud de Arco de Meridiano

$$S_m = a(1-e^2)\left[\boldsymbol{A}(\varphi_2-\varphi_1) - \frac{\boldsymbol{B}}{2}(sen2\varphi_2 - sen2\varphi_1) + \frac{\boldsymbol{C}}{4}(sen4\varphi_2 - sen4\varphi_1) - \frac{\boldsymbol{D}}{6}(sen6\varphi_2 - sen6\varphi_1) + \frac{\boldsymbol{E}}{8}(sen8\varphi_2 - sen8\varphi_1) - \frac{\boldsymbol{F}}{10}(sen10\varphi_2 - sen10\varphi_1)\right]$$

Fuente: www.thegreenwichmeridian.org



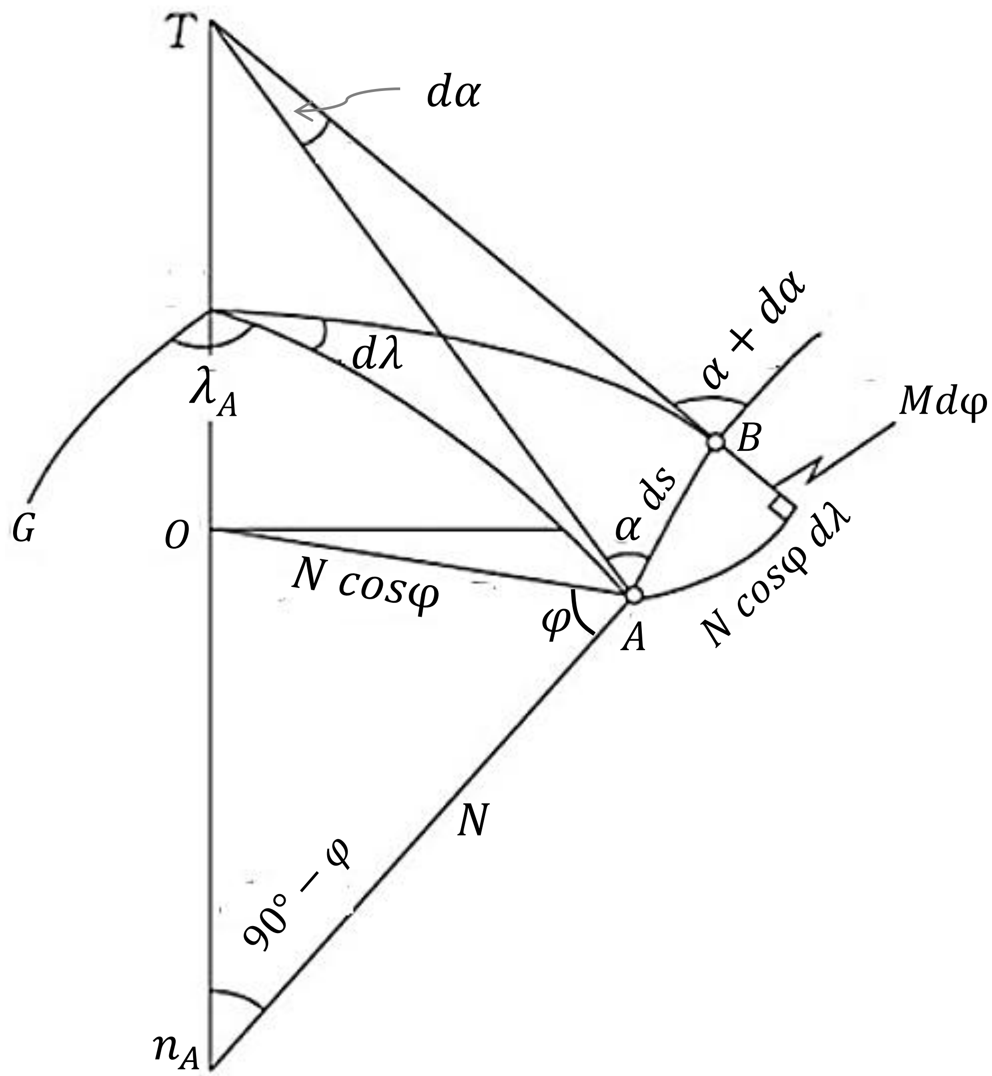
Longitud de Arco de Paralelo

$$S_p = N\,cos\varphi\,\Delta\lambda$$

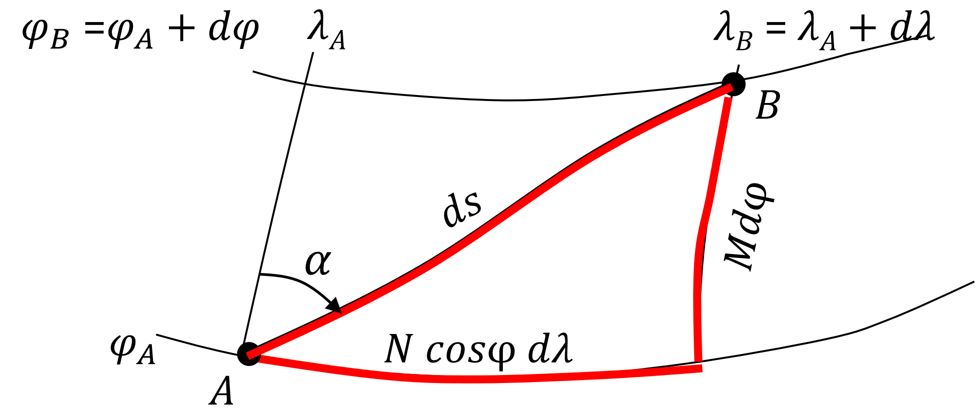
(7) Rapp, Richard H. (1991). *Geometric geodesy part I*. Department of Geodetic Science and Surveying, Ohio State University.
(9) Jekeli, C. (2006). *Geometric reference systems in geodesy*. Division of Geodetic Science, School of Earth Sciences. Ohio State University.
(10) Jordan, W. (1962). *Handbook of Geodesy* (Vol. 3). Corps of Engineers, United States Army, Army Map Service

Geodesia

Ecuación Fundamental de la Geodésica[7,9,10]:



Fuente: Lu Z., Qu Y., Qiao S. (2014)



$$\cos\alpha = \frac{M d\varphi}{dS} \rightarrow dS\cos\alpha = M d\varphi$$

$$sen\,\alpha = \frac{N\,cos\varphi\,d\lambda}{dS} \rightarrow dS\,sen\,\alpha = N\,cos\varphi\,d\lambda$$

$$\left.\begin{aligned} d\alpha &= \frac{N\,cos\varphi\,d\lambda}{AT} \\ \cos(90° - \varphi) &= \frac{N\,cos\varphi}{AT} \end{aligned}\right\} \quad d\alpha = sen\,\varphi\,d\lambda \rightarrow d\alpha = \frac{sen\,\alpha}{N} tg\varphi\,dS$$

Ecuación de Bessel

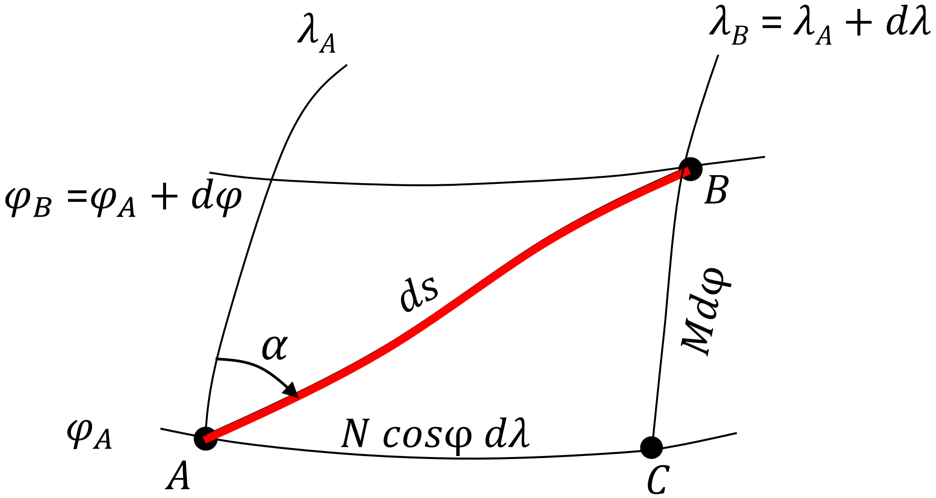
(7) Rapp, Richard H. (1991). ***Geometric geodesy part I.*** **Department of Geodetic Science and Surveying, Ohio State University.**
(9) Jekeli, C. (2006). ***Geometric reference systems in geodesy*****. Division of Geodetic Science, School of Earth Sciences. Ohio State University.**
(10) Jordan, W. (1962). ***Handbook of Geodesy*** **(Vol. 3). Corps of Engineers, United States Army, Army Map Service**

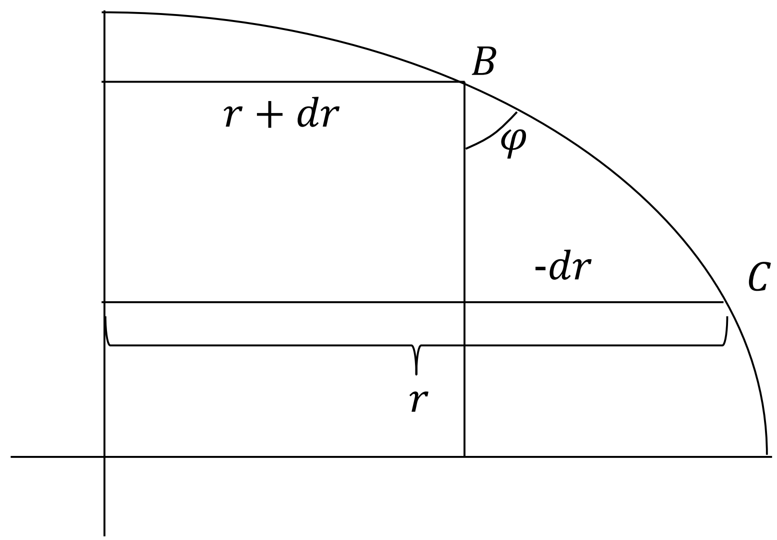
Geodesia

Ecuación Fundamental de la Geodésicas(7,9,10):

$$dS = \sqrt{(Md\varphi)^2 + (N\cos\varphi\, d\lambda)^2} \;\rightarrow\; I = \int_A^B dS \rightarrow \text{mínima}$$

Longitud más corta entre dos puntos sobre una superficie.





$$-dr = M\, sen\varphi\, d\varphi$$

$$\left.\begin{array}{l} dS\,\cos\alpha = Md\varphi \\ dS\,\,sen\,\alpha = N\cos\varphi\, d\lambda = r\, d\lambda \end{array}\right\}$$

$$\left.\begin{array}{l} \cos\alpha = \dfrac{Md\varphi}{dS} \rightarrow\; r\,d\alpha\,\cos\alpha = \dfrac{Md\varphi}{dS} r\,d\alpha = \dfrac{Md\varphi}{dS} r\,sen\varphi\,d\lambda \\ sen\,\alpha = \dfrac{N\cos\varphi\,d\lambda}{dS} \rightarrow\; dr\,\,sen\,\alpha = r\dfrac{d\lambda}{dS} dr = -r\dfrac{d\lambda}{dS}\, M\,sen\varphi\,d\varphi \end{array}\right\} \; r\,d\alpha\,\cos\alpha + dr\,\,sen\,\alpha = 0$$

$$\downarrow$$

$$r\,\,sen\,\alpha = cte$$

$$N\cos\varphi\,\,sen\,\alpha = cte$$

Ecuación de Clairaut

(7) Rapp, Richard H. (1991). *Geometric geodesy part I*. Department of Geodetic Science and Surveying, Ohio State University.
(9) Jekeli, C. (2006). *Geometric reference systems in geodesy*. Division of Geodetic Science, School of Earth Sciences. Ohio State University.
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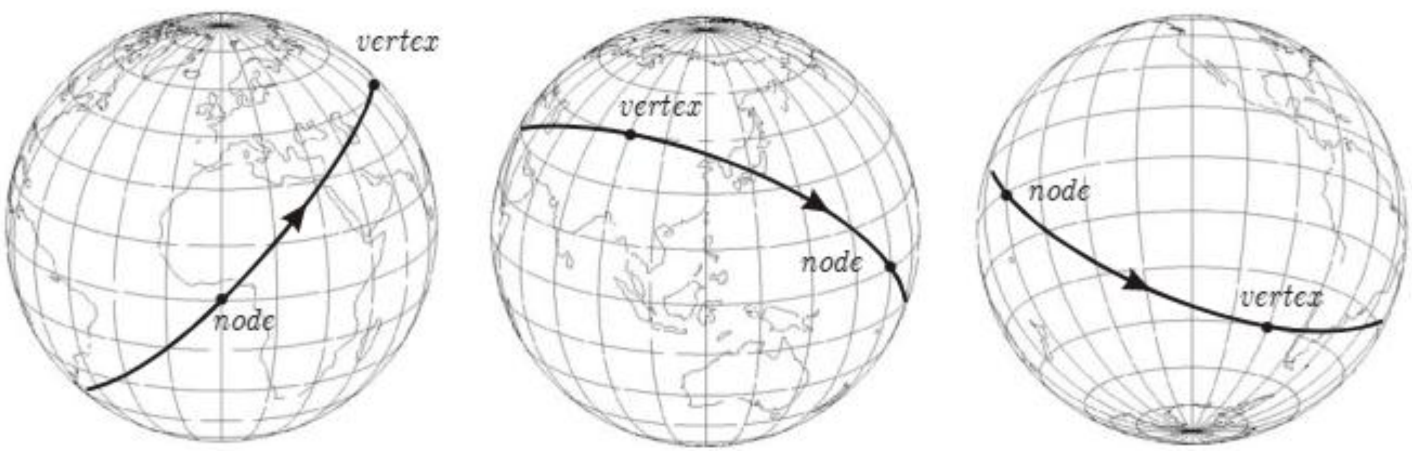
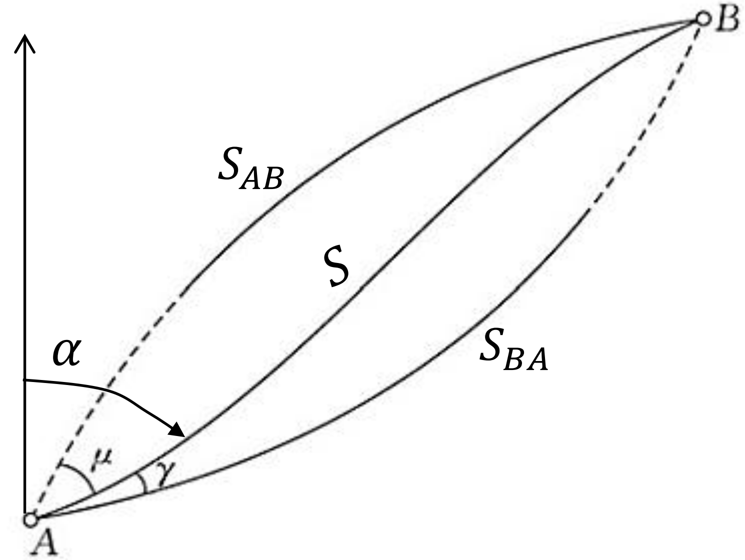
Geodesia

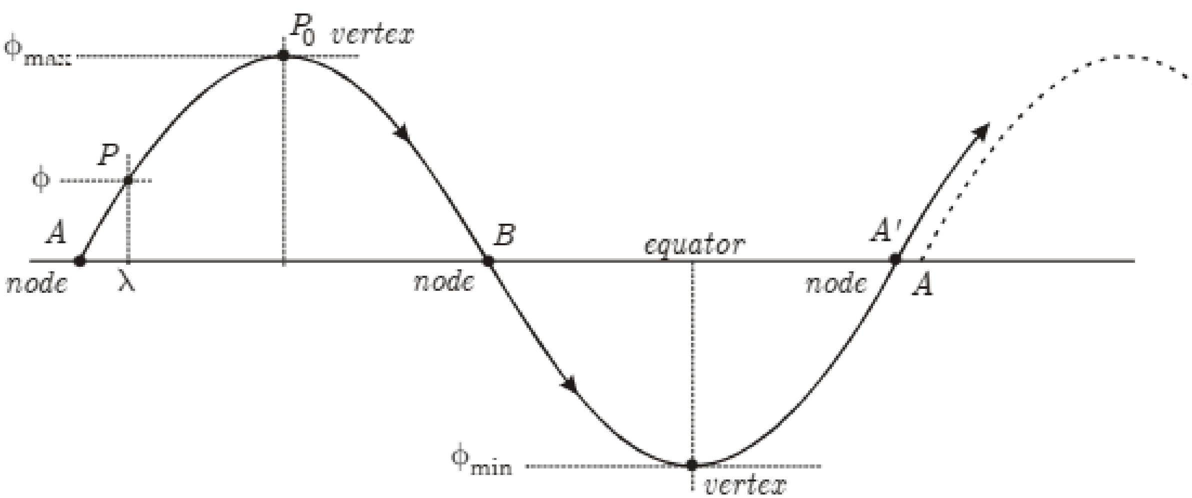
Geodésicas[7,9,10]: Longitud más corta entre dos puntos sobre una superficie.

Ecuación de Clairaut $N\ cos\varphi\ sen\ \alpha = cte$





Fuente: Lu Z., Qu Y., Qiao S. (2014)



$N\ cos\varphi = a\ cos\beta$

$a\ cos\beta\ \ sen\ \alpha = k$

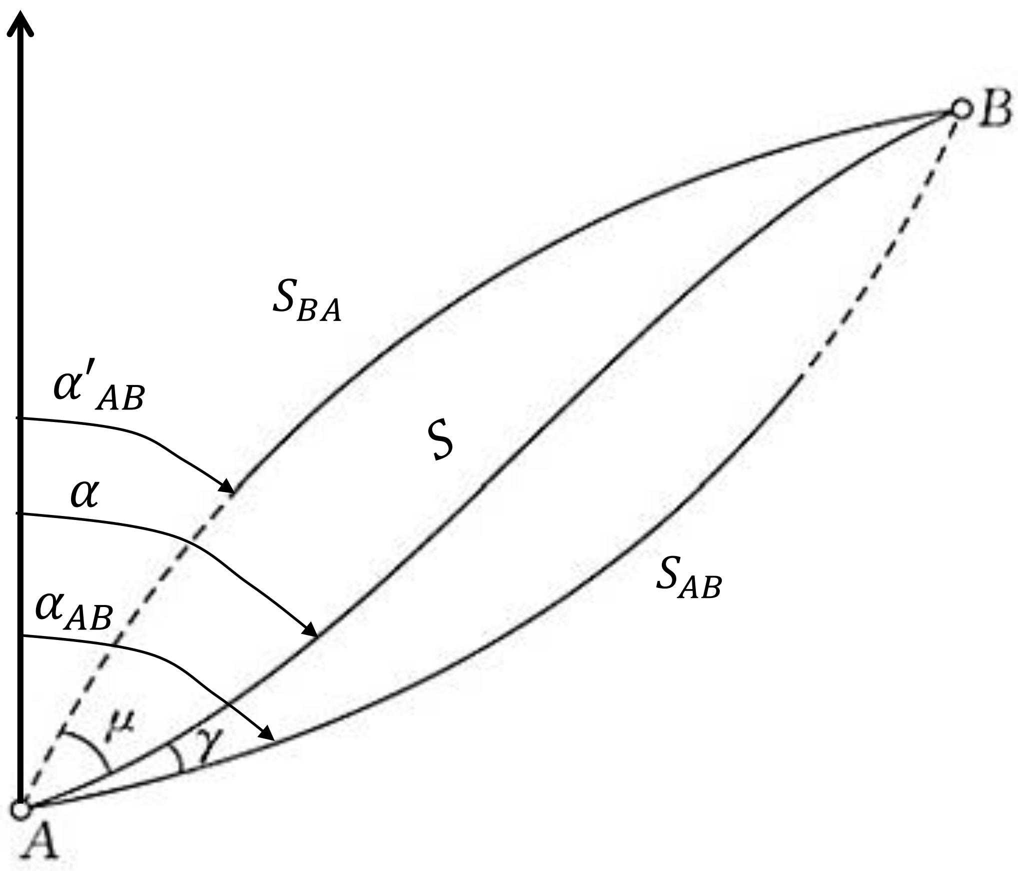
Sí β=0° $\rightarrow$ $sen\ \alpha_E = \dfrac{k}{a}$

Fuente: Deakin, R. E., & Hunter, M. N. (2009).

(7) Rapp, Richard H. (1991). ***Geometric geodesy part I*****. Department of Geodetic Science and Surveying, Ohio State University.**
(9) Jekeli, C. (2006). ***Geometric reference systems in geodesy*****. Division of Geodetic Science, School of Earth Sciences. Ohio State University.**
(10) Jordan, W. (1962). ***Handbook of Geodesy*** **(Vol. 3). Corps of Engineers, United States Army, Army Map Service**

Geodesia

Diferencia entre sección normal y Geodésica[7,9,10]:



Fuente: Lu Z., Qu Y., Qiao S. (2014)

Diferencia de azimut entre secciones recíprocas

$$\Delta = \alpha_{AB} - \alpha'_{AB} = \frac{e^2}{4}\left(\frac{S_{AB}}{N_A}\right)^2 sen2\alpha_{AB}\, cos^2\varphi_m$$

Diferencia de azimut entre sección normal y geodésica

$$\gamma = \alpha_{AB} - \alpha = \frac{\eta^2}{2}\left(\frac{S_{AB}}{N_A}\right)^2 sen\alpha_{AB}\, cos\alpha_{AB} + \frac{\eta^2 t}{24}\left(\frac{S_{AB}}{N_A}\right)^3 sen\alpha_{AB} \cong \frac{\Delta}{3}$$

Diferencia de longitud entre sección normal y geodésica

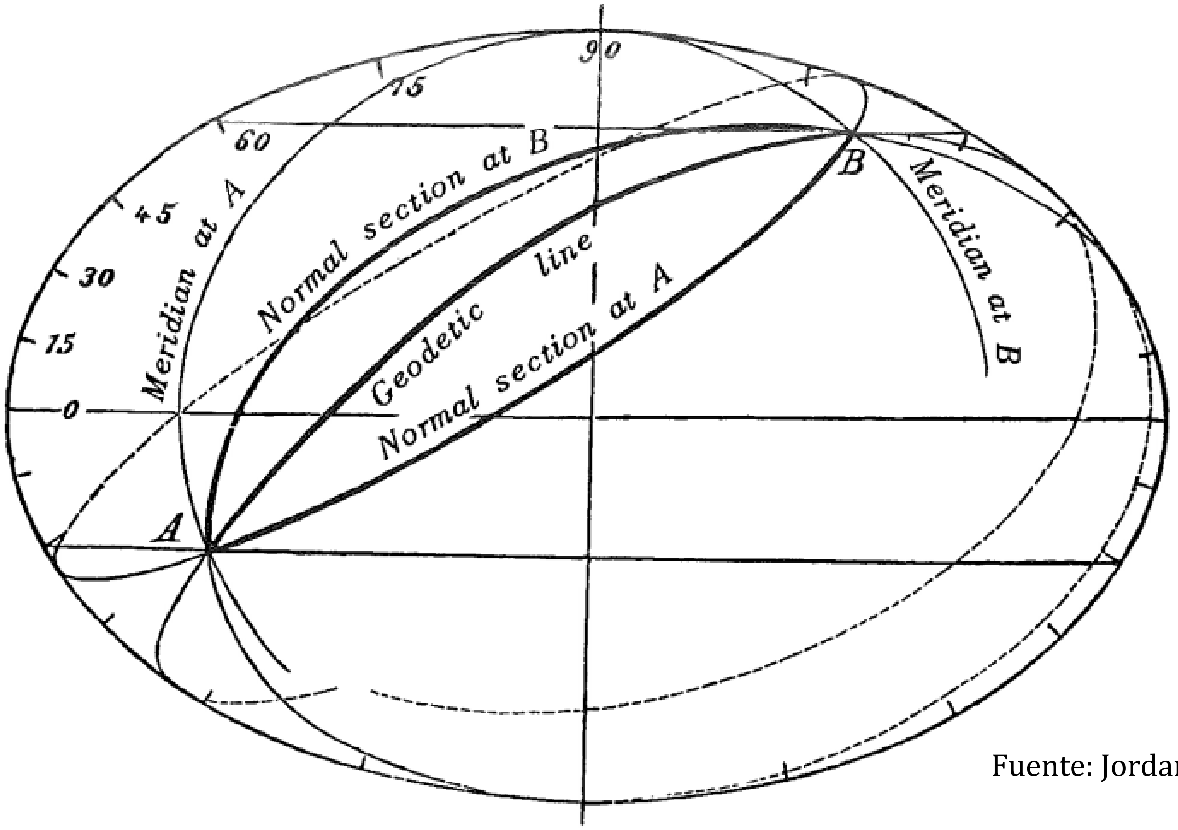
$$S_{AB} - S = \frac{a\, e^4}{360}\left(\frac{S_{AB}}{N_A}\right)^5 sen^2 2\alpha\, cos^4\varphi$$

(7) Rapp, Richard H. (1991). ***Geometric geodesy part I*****. Department of Geodetic Science and Surveying, Ohio State University.**
(9) Jekeli, C. (2006). ***Geometric reference systems in geodesy*****. Division of Geodetic Science, School of Earth Sciences. Ohio State University.**
(10) Jordan, W. (1962). ***Handbook of Geodesy*** **(Vol. 3). Corps of Engineers, United States Army, Army Map Service**

Geodesia

Consecuencias[7,9,10]:

- La normal principal de la geodésica es normal al elipsoide en cada punto de la geodésica;
- Cualquier meridiano es una geodésica;
- El ecuador es una geodésica, excepto para puntos diametralmente opuestos;
- Los círculos paralelos no son geodésicas;
- Una geodésica en el elipsoide es una curva de doble curvatura, excepto dos casos particulares;
- Una geodésica alcanza latitudes máximas y mínimas ($\varphi_{máx} = -\varphi_{mín}$);



Fuente: Jordan, W. (1962).

(7) Rapp, Richard H. (1991). *Geometric geodesy part I*. Department of Geodetic Science and Surveying, Ohio State University.
(9) Jekeli, C. (2006). *Geometric reference systems in geodesy*. Division of Geodetic Science, School of Earth Sciences. Ohio State University.
(10) Jordan, W. (1962). *Handbook of Geodesy* (Vol. 3). Corps of Engineers, United States Army, Army Map Service