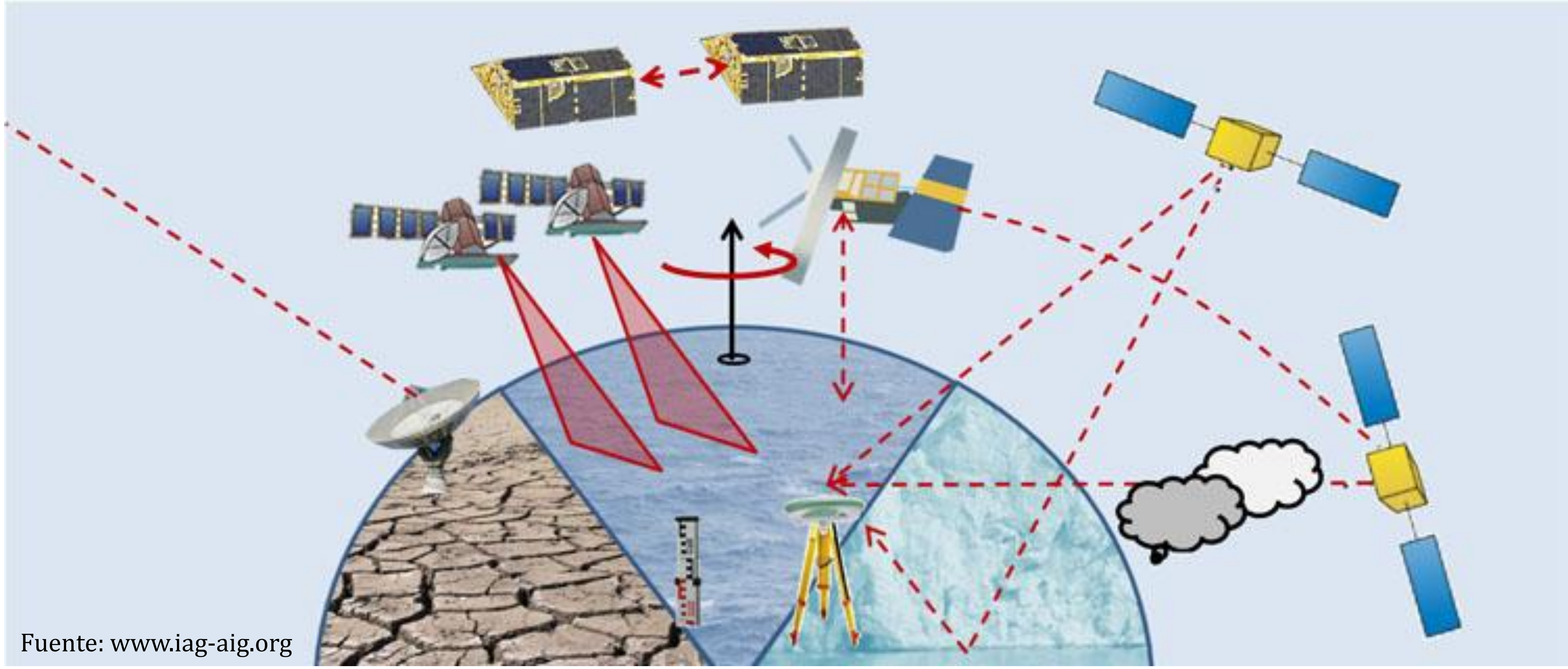
GEODESIA, PERO… ¿PARA QUÉ?



Fuente: www.iag-aig.org

Transporte de coordenadas[11]:

Cálculo de elementos de un triángulo

Cantidades observadas $\rightarrow$ $a, \beta, \gamma, \overline{AB}$

$$\left.\begin{aligned} &\frac{sen\alpha}{sen\gamma} = \frac{a}{\overline{AB}} \\ &b^2 = a^2 + \overline{AB}^2 - 2a\overline{AB}\cos\beta \end{aligned}\right\}$$

Obtención de lados y ángulos de ABP

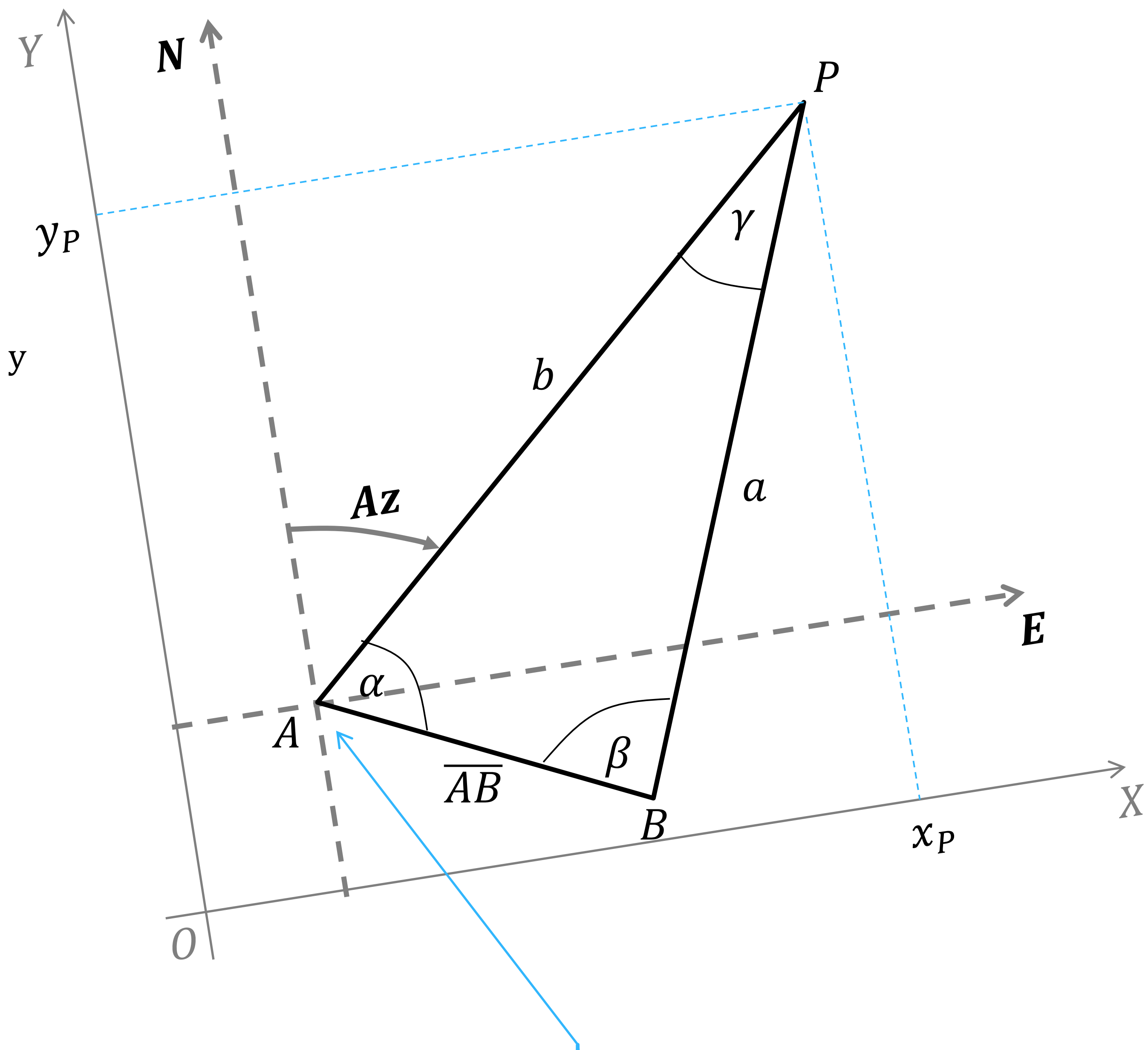
Agregamos otras cantidades $\rightarrow$ $Az, (x, y)_A$

$$x_P = b\, senAz + x_A$$
$$y_P = b\, cosAz + y_A$$

Cantidades observadas $\rightarrow$ $(x, y)_A; (x, y)_P$

$$tgAz = \frac{x_P - x_A}{y_P - y_A}$$

$$b = \frac{x_P - x_A}{senAz} = \frac{y_P - y_A}{cosAz}$$



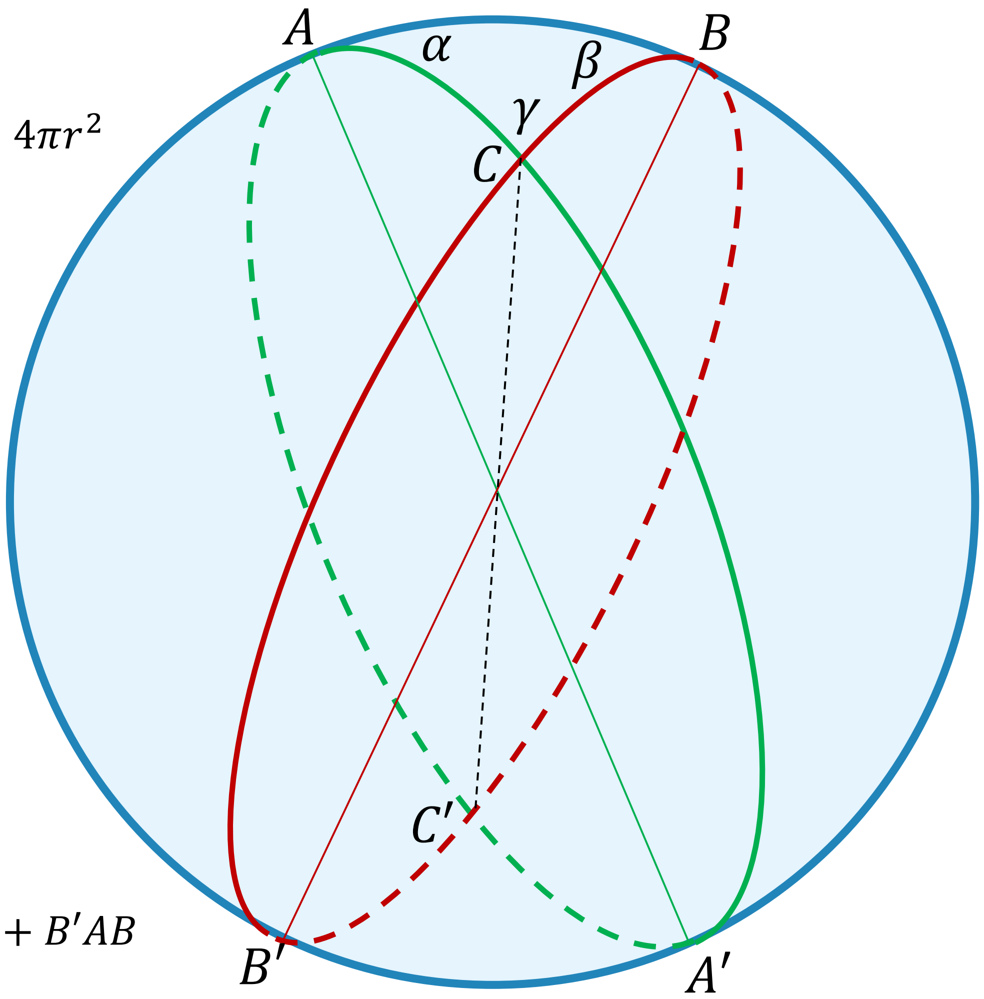
Punto Fundamental Astronómico $\rightarrow$ Φ, Λ, Az

(11) **Domínguez García-Tejero, F. (1998).** ***Topografía general y aplicada*****.**

Geodesia

Solución de triángulos elipsoidales[7,9,10]:

$Sup = 4\pi r^2$



Exceso esférico → $\varepsilon = (\alpha + \beta + \gamma) - 180°$

Teorema fundamental → $\varepsilon = \frac{ST}{R^2}$

$$\left.\begin{aligned} Sup\ AA' &= \frac{\alpha}{360}(4\pi r^2) \\ Sup\ BB' &= \frac{\beta}{360}(4\pi r^2) \\ Sup\ CC' &= \frac{\gamma}{360}(4\pi r^2) \end{aligned}\right\} \quad Sup\ AA' + Sup\ BB' + Sup\ CC' = \frac{\alpha + \beta + \gamma}{360}(4\pi r^2)$$

$$\left.\begin{aligned} Sup\ AA' &= ST + A'BC \\ Sup\ BB' &= ST + B'AC \\ Sup\ CC' &= ST + B'AB \end{aligned}\right\} \quad Sup\ AA' + Sup\ BB' + Sup\ CC' = 3\ ST + A'BC + B'AC + B'AB$$

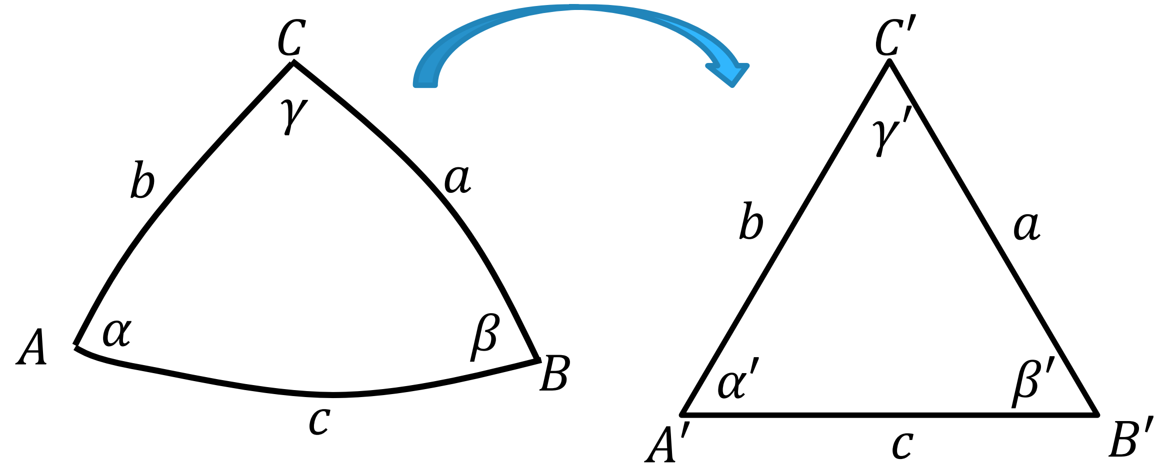
$$\left.\begin{aligned} 3\ ST &= 2\ ST + ST \\ ST + A'BC + B'AC + B'AB &= 2\pi r^2 \end{aligned}\right\} \quad Sup\ AA' + Sup\ BB' + Sup\ CC' = 2\ ST + 2\pi r^2$$

$$\frac{\alpha + \beta + \gamma}{360}(4\pi r^2) = 2\ ST + 2\pi r^2 \longrightarrow ST = (\alpha + \beta + \gamma - 180°)\frac{\pi}{180°}r^2$$

(7) Rapp, Richard H. (1991). *Geometric geodesy part I*. Department of Geodetic Science and Surveying, Ohio State University.
(9) Jekeli, C. (2006). *Geometric reference systems in geodesy*. Division of Geodetic Science, School of Earth Sciences. Ohio State University.
(10) Jordan, W. (1962). *Handbook of Geodesy* (Vol. 3). Corps of Engineers, United States Army, Army Map Service

Geodesia

Solución de triángulos elipsoidales[7,9,10]:



Teorema de Legendre: se asumen lados del triángulo esférico y del triángulo auxiliar iguales, entonces los ángulos del triángulo auxiliar varían en un tercio del exceso esférico.

$$\cos\frac{a}{R} = \cos\frac{b}{R}\cos\frac{c}{R} + sen\frac{b}{R}\,sen\frac{c}{R}\cos\alpha$$

$$a^2 = b^2 + c^2 - 2bc\cos\alpha'$$

$$\left.\begin{aligned} \alpha - \alpha' &= \frac{p}{3R^2} \\ \beta - \beta' &= \frac{p}{3R^2} \\ \gamma - \gamma' &= \frac{p}{3R^2} \end{aligned}\right\} \longrightarrow \alpha + \beta + \gamma = 180° + \frac{p}{R^2} \left\{\begin{aligned} \alpha - \alpha' &= \frac{\varepsilon}{3} \\ \beta - \beta' &= \frac{\varepsilon}{3} \\ \gamma - \gamma' &= \frac{\varepsilon}{3} \end{aligned}\right.$$

Aproximación esférica

$$\varepsilon = \frac{p}{R^2}\left(1 + \frac{a^2+b^2+c^2}{24R^2}\right)$$

$$p = \sqrt{s(s-a)(s-b)(s-c)}$$

$$s = \frac{a+b+c}{2}$$

$$m^2 = \frac{a^2+b^2+c^2}{3}$$

$$\left.\right\} \begin{aligned} \alpha - \alpha' &= \frac{\varepsilon}{3} + \frac{\varepsilon}{60R^2}(m^2 - a^2) \\ \beta - \beta' &= \frac{\varepsilon}{3} + \frac{\varepsilon}{60R^2}(m^2 - b^2) \\ \gamma - \gamma' &= \frac{\varepsilon}{3} + \frac{\varepsilon}{60R^2}(m^2 - c^2) \end{aligned}$$

Aproximación elipsóidica

$$K_A = (MN)_A{}^{-1};\ K_B = (MN)_B{}^{-1};\ K_C = (MN)_C{}^{-1}$$

$$K_m = (K_A + K_B + K_A)/3 \qquad \varepsilon = pK_m\left(1 + \frac{m^2K_m}{8}\right)$$

$$\alpha - \alpha' = \frac{\varepsilon}{3} + \frac{\varepsilon}{60}K_m(m^2 - a^2) + \frac{\varepsilon}{12}\frac{K_A - K_m}{K_m}$$

$$\beta - \beta' = \frac{\varepsilon}{3} + \frac{\varepsilon}{60}K_m(m^2 - b^2) + \frac{\varepsilon}{12}\frac{K_B - K_m}{K_m}$$

$$\gamma - \gamma' = \frac{\varepsilon}{3} + \frac{\varepsilon}{60}K_m(m^2 - c^2) + \frac{\varepsilon}{12}\frac{K_C - K_m}{K_m}$$

(7) Rapp, Richard H. (1991). *Geometric geodesy part I*. Department of Geodetic Science and Surveying, Ohio State University.
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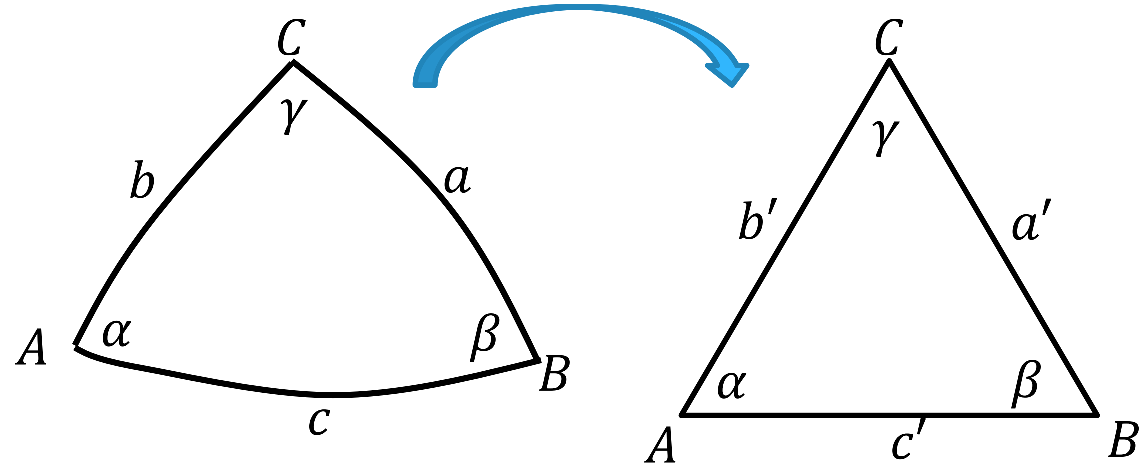
(10) Jordan, W. (1962). *Handbook of Geodesy* (Vol. 3). Corps of Engineers, United States Army, Army Map Service

Geodesia

Solución de triángulos elipsoidales[7,9,10]:

Cantidades adicionales: cantidad que modifica los lados del triángulo esférico.



$$\left.\begin{array}{l}\dfrac{\operatorname{sen}\alpha}{\operatorname{sen}\beta}=\dfrac{\operatorname{sen}\frac{a}{R}}{\operatorname{sen}\frac{b}{R}}\\[2ex] \dfrac{\operatorname{sen}\alpha}{\operatorname{sen}\beta}=\dfrac{a'}{b'}\end{array}\right\} \quad \frac{a'}{b'}=\frac{\operatorname{sen}\frac{a}{R}}{\operatorname{sen}\frac{b}{R}}=\frac{a-\frac{a^3}{6R^2}}{b-\frac{b^3}{6R^2}} \longrightarrow \left\{\begin{array}{l} a'=a-\boxed{\dfrac{a^3}{6R^2}}\\[2ex] b'=b-\boxed{\dfrac{b^3}{6R^2}}\end{array}\right. \longrightarrow l'=l-\frac{l^3}{6R^2}$$

aditamentos

$$\left.\begin{array}{l} b'=a'\dfrac{\operatorname{sen}\beta}{\operatorname{sen}\alpha}\\[2ex] b=b'+\dfrac{b^3}{6R^2}\end{array}\right\} \quad b=a'\frac{\operatorname{sen}\beta}{\operatorname{sen}\alpha}+\frac{b^3}{6R^2}\approx b'+\frac{b'^3}{6R^2}$$

$$c'=a'\frac{\operatorname{sen}\gamma}{\operatorname{sen}\alpha}\longrightarrow c=a'\frac{\operatorname{sen}\gamma}{\operatorname{sen}\alpha}+\frac{c^3}{6R^2}\approx c'+\frac{c'^3}{6R^2}$$

(7) Rapp, Richard H. (1991). *Geometric geodesy part I*. Department of Geodetic Science and Surveying, Ohio State University.
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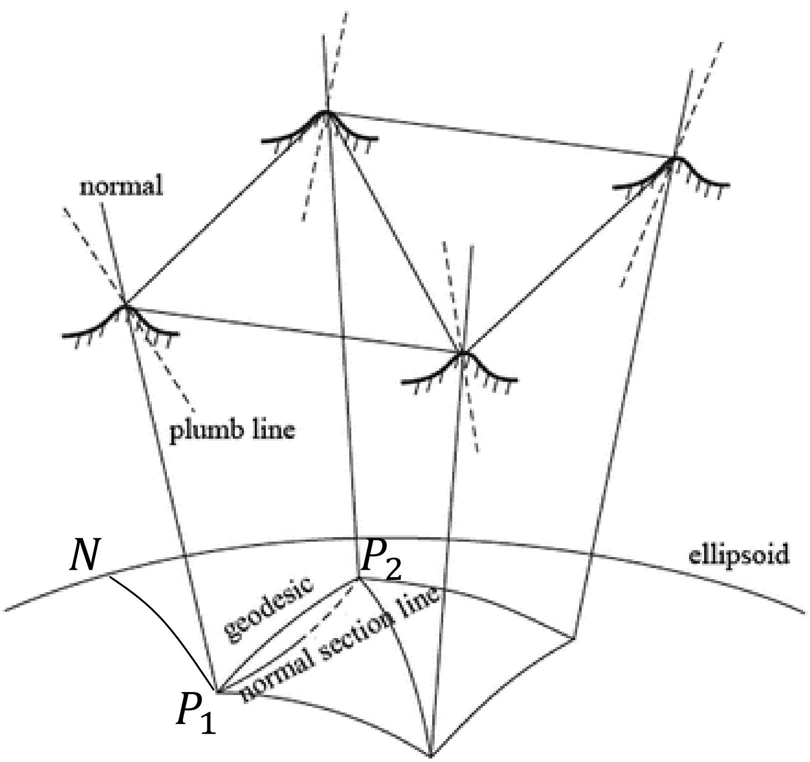
Problema Fundamental de la Geodesia(7,9,10):

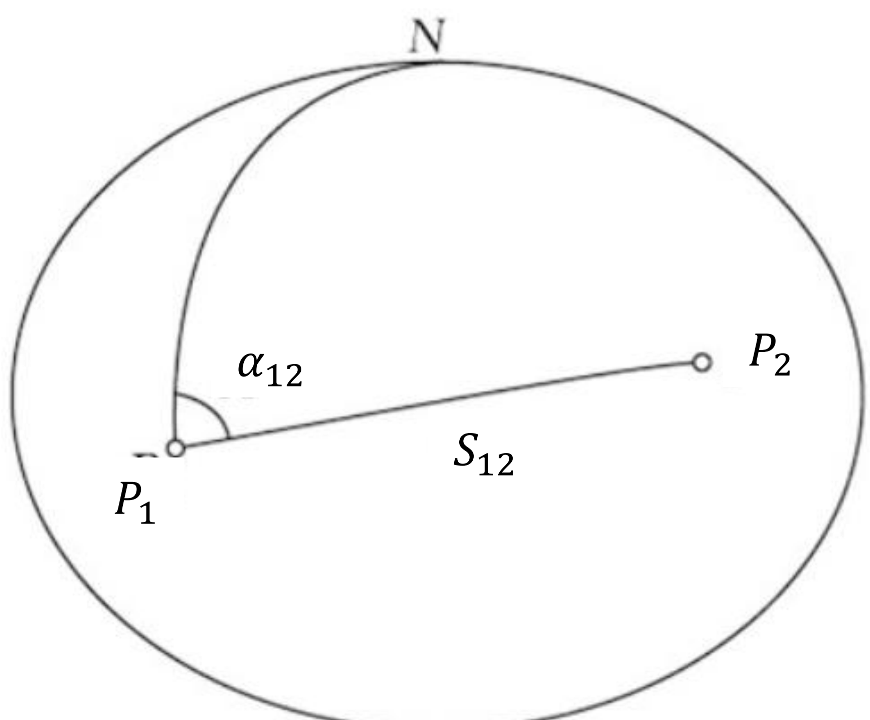
Cálculo de coordenadas, direcciones y distancias sobre el elipsoide

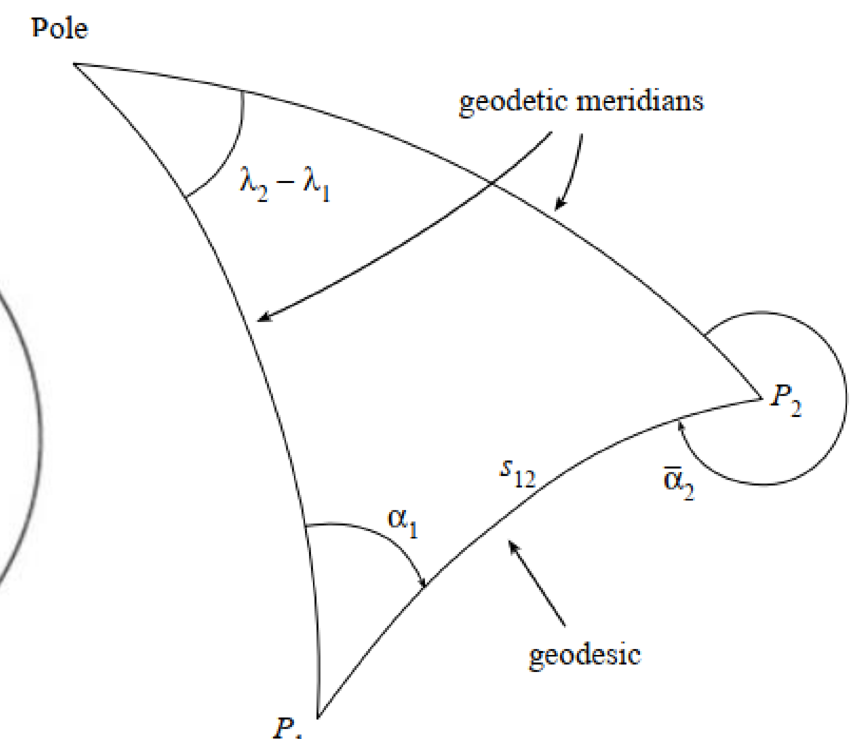
PFG
- Problema Geodésico Directo (PGD) $\longrightarrow$ $\varphi_1, \lambda_1, \alpha_1, S_{12} \rightarrow \varphi_2, \lambda_2, \bar{\alpha}_2$
- Problema Geodésico Inverso (PGI) $\longrightarrow$ $\varphi_1, \lambda_1, \varphi_2, \lambda_2 \rightarrow \alpha_1, \bar{\alpha}_2, S_{12}$

Líneas cortas ($S < 200\ Km$)
Líneas medias (200 Km $< S < 1000\ Km$)
Líneas largas (1000 Km $< S < 2000\ Km$)



Fuente: Lu Z., Qu Y., Qiao S. (2014)





Fuente: Jekeli, C. (2006)

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(9) Jekeli, C. (2006). *Geometric reference systems in geodesy*. Division of Geodetic Science, School of Earth Sciences. Ohio State University.
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Geodesia

Problema Fundamental de la Geodesia[7,9,10]:

Cálculo de coordenadas, direcciones y distancias sobre el elipsoide

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- Líneas cortas ($S < 200\ Km$)
- Líneas medias (200 Km $< S < 1000\ Km$)
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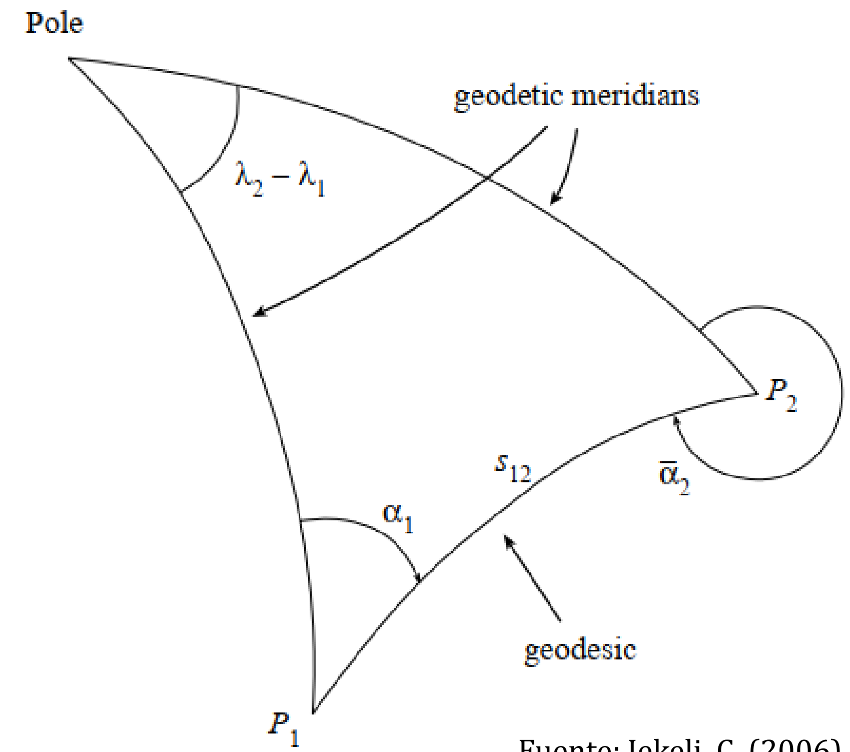
Soluciones
- Desarrollos en serie e iteraciones
- Esfera auxiliar
- Integración numérica



Fuente: Jekeli, C. (2006)

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Geodesia

Problema Fundamental de la Geodesia[7,9,10]:

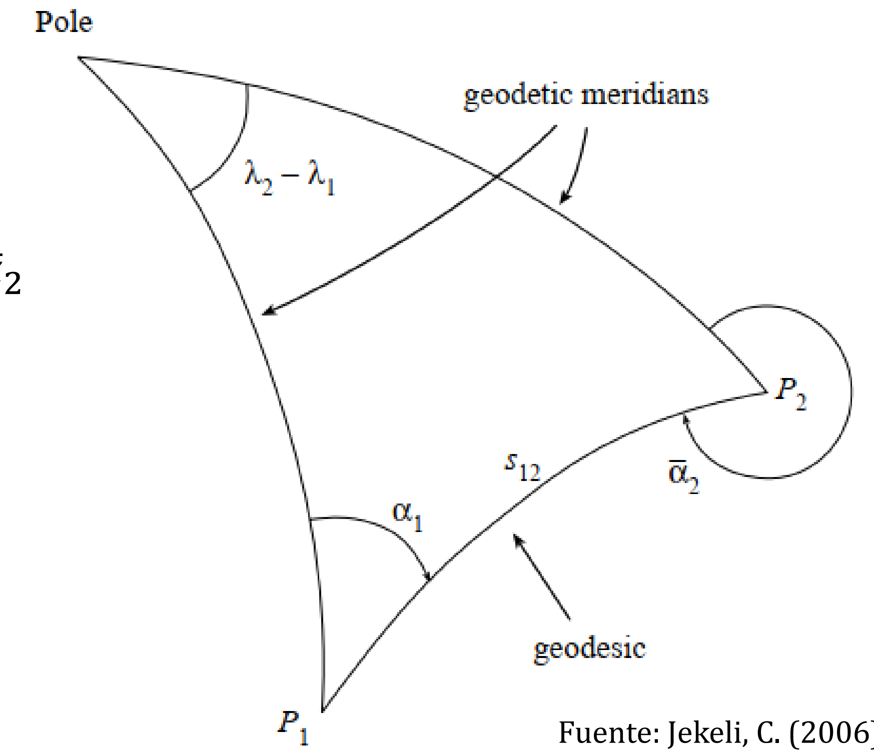
Problema Geodésico Directo (PGD)
Series de Legendre

→ $\varphi_1, \lambda_1, \alpha_1, S_{12} \rightarrow \varphi_2, \lambda_2, \bar{\alpha}_2$

$$\varphi = \varphi(S), \qquad \lambda = \lambda(S), \qquad \alpha = \alpha(S)$$

$$\begin{cases} \varphi_2 - \varphi_1 = \left.\dfrac{d\varphi}{dS}\right|_1 S_{12} + \dfrac{1}{2!}\left.\dfrac{d^2\varphi}{dS^2}\right|_1 S^2{}_{12} + \ \dots \\ \lambda_2 - \lambda_1 = \left.\dfrac{d\lambda}{dS}\right|_1 S_{12} + \dfrac{1}{2!}\left.\dfrac{d^2\lambda}{dS^2}\right|_1 S^2{}_{12} + \ \dots \\ (\bar{\alpha}_2 - \alpha_1) - \pi = \left.\dfrac{d\alpha}{dS}\right|_1 S_{12} + \dfrac{1}{2!}\left.\dfrac{d^2\alpha}{dS^2}\right|_1 S^2{}_{12} + \ \dots \end{cases}$$

$$\begin{cases} \cos\alpha = \dfrac{M d\varphi}{dS} \rightarrow \dfrac{d\varphi}{dS} = \dfrac{\cos\alpha}{M} \\ sen\,\alpha = \dfrac{N\,cos\varphi\,d\lambda}{dS} \rightarrow \dfrac{d\lambda}{dS} = \dfrac{sen\,\alpha}{N\,cos\varphi} \\ d\alpha = sen\,\varphi\,d\lambda \rightarrow \dfrac{d\alpha}{dS} = \dfrac{sen\,\alpha}{N\,cos\varphi} \end{cases}$$



Fuente: Jekeli, C. (2006)

Algunas variables útiles:

$$V^2 = 1 + \eta^2$$
$$\eta^2 = e'^2 cos^2\varphi_1$$
$$t = tg\varphi_1$$
$$u = \frac{S_{12}\ sen\alpha_1}{N_1}$$
$$v = \frac{S_{12} cos\alpha_1}{N_1}$$

$$\rho^\circ = \frac{180^\circ}{\pi}$$
$$\rho'' = \frac{180^\circ}{\pi} 3600$$

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(9) Jekeli, C. (2006). *Geometric reference systems in geodesy*. Division of Geodetic Science, School of Earth Sciences. Ohio State University.
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Geodesia

Problema Geodésico Directo(7,9,10): → $\varphi_1, \lambda_1, \alpha_1, S_{12} \rightarrow \varphi_2, \lambda_2, \bar{\alpha}_2$

$$\frac{\varphi_2-\varphi_1}{V^2} = u - \frac{1}{2\rho}v^2 t - \frac{3}{2\rho}u^2v^2t - \frac{v^2u}{6\rho^2}(1+3t^2+\eta^2-9\eta^2t^2) - \frac{u^3}{2\rho^2}\eta^2(1-t^2)$$
$$+\frac{v^4t}{24\rho^3}(1+3t^2+\eta^2-9\eta^2t^2) - \frac{v^2u^2}{12\rho^3}t(4+6t^2-13\eta^2-9\eta^2t^2) + \frac{u^4}{2\rho^3}\eta^2 t$$
$$+\frac{v^4u}{120\rho^4}(1+30t^2+45t^4) - \frac{v^2u^3}{30\rho^4}(2+15t^2+15t^4)$$

$$(\lambda_2-\lambda_1)cos\varphi_1 = v + \frac{1}{\rho}vut - \frac{v^3t^2}{3\rho^2} + \frac{u^2v}{3\rho^2}(1+3t^2+\eta^2)$$
$$-\frac{v^3u\,t}{3\rho^3}(1+3t^2+\eta^2) + \frac{u^3v\,t}{3\rho^3}(2+3t^2+\eta^2)$$
$$+\frac{v^5t^2}{15\rho^4}(1+3t^2) + \frac{u^4v}{15\rho^4}(2+15t^2+15t^4) - \frac{v^3u^2}{15\rho^4}(1+20t^2+30t^4)$$

$$(\bar{\alpha}_2-\alpha_1) - \pi = vt + \frac{u\,v}{2\rho}(1+2t^2+\eta^2) - \frac{v^3t}{6\rho^2}(1+2t^2+\eta^2) + \frac{u^2v\,t}{6\rho^2}(5+6t^2+\eta^2-4\eta^2)$$
$$-\frac{v^3u}{24\rho^3}(1+20t^2+24t^4+2\eta^2+8\eta^2t^2) + \frac{u^3v}{24\rho^3}(5+28t^2+24t^4+6\eta^2+8\eta^2t^2)$$
$$+\frac{v^5t}{120\rho^4}(1+20t^2+24t^4) - \frac{v^3u^2}{120\rho^4}(58+280t^2+240t^4) + \frac{vu^4}{120\rho^4}(61+180t^2+120t^4)$$

(7) Rapp, Richard H. (1991). *Geometric geodesy part I*. Department of Geodetic Science and Surveying, Ohio State University.
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Geodesia

Problema Geodésico Inverso[7,9,10]: → $\varphi_1, \lambda_1, \varphi_2, \lambda_2 \rightarrow \alpha_1, \bar{\alpha}_2, S_{12}$

$$\frac{\varphi_2 - \varphi_1}{V^2} = u - \frac{1}{2\rho} v^2 t - \ \dots \longrightarrow \Delta\varphi = \varphi_2 - \varphi_1 = V^2 u + \delta\varphi \longrightarrow u = \frac{\Delta\varphi - \delta\varphi}{V^2}$$

$$(\lambda_2 - \lambda_1) cos\varphi_1 = v + \frac{1}{\rho} vut - \ \dots \longrightarrow \Delta\lambda = \lambda_2 - \lambda_1 = \frac{v}{cos\varphi_1} + \delta\lambda \longrightarrow v = (\Delta\lambda - \delta\lambda) cos\varphi_1$$

$$\alpha_1 = tg^{-1}\left(V^2 \frac{\Delta\lambda - \delta\lambda}{\Delta\varphi - \delta\varphi} cos\varphi_1\right)$$

$$\left\{\begin{array}{l} S_{12} = \dfrac{N_1\, cos\varphi_1}{sen\alpha_1}(\Delta\lambda - \delta\lambda) \\ S_{12} = \dfrac{N_1}{cos\alpha_1}\dfrac{\Delta\varphi - \delta\varphi}{V^2} \end{array}\right. \qquad \alpha_1 = ?$$

Con: $\delta\varphi = 0$; $\delta\lambda = 0$

$$\left.\begin{array}{l} u^0 = \dfrac{\Delta\varphi}{V^2} \\ v^0 = (\Delta\lambda) cos\varphi_1 \end{array}\right\} \begin{array}{l} \alpha_1{}^0 = tg^{-1}\left(V^2 \dfrac{\Delta\lambda}{\Delta\varphi} cos\varphi_1\right) \\ S_{12}{}^0 = \dfrac{N_1\, cos\varphi_1}{sen\alpha_1}(\Delta\lambda) \end{array}$$

Algunas variables útiles:

$$V^2 = 1 + \eta^2$$

$$\eta^2 = e'^2 cos^2\varphi_1$$

$$t = tg\varphi_1$$

$$u = \frac{S_{12}\, sen\alpha_1}{N_1}$$

$$v = \frac{S_{12} cos\alpha_1}{N_1}$$

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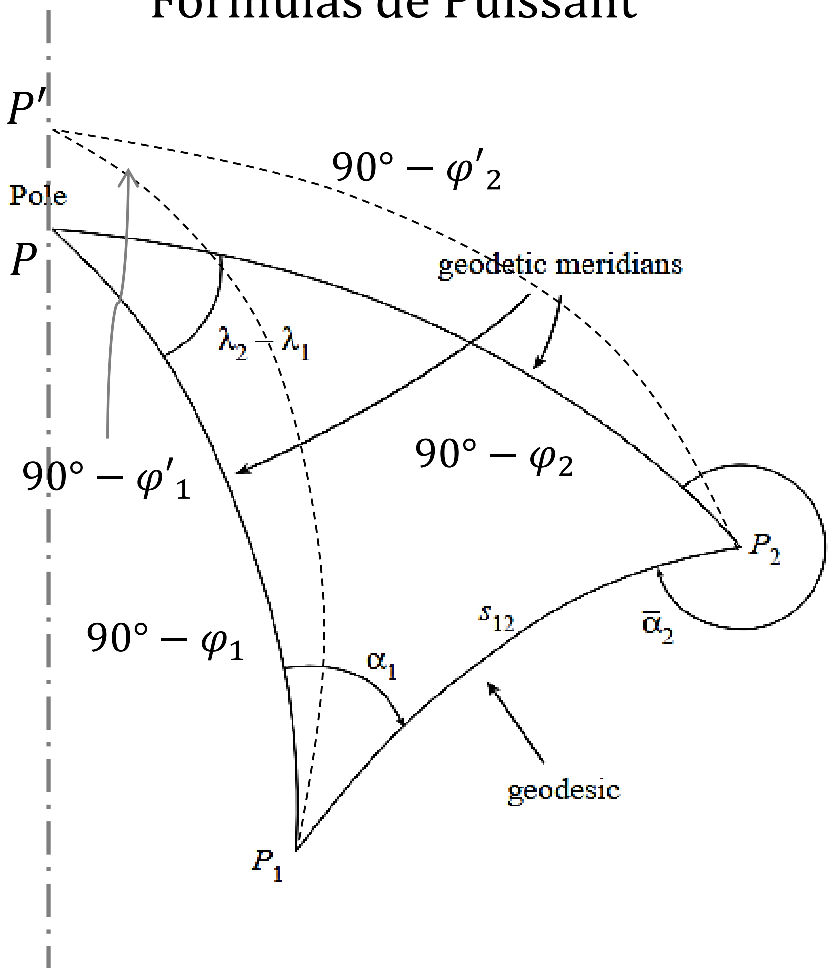
Geodesia

Problema Geodésico Directo[7,9,10]**:** → $\varphi_1, \lambda_1, \alpha_1, S_{12} \rightarrow \varphi_2, \lambda_2, \bar{\alpha}_2$

Fórmulas de Puissant



Esfera de radio N_1, tangente en P_1

Casi coincidente en P_2 (si $S < 100\ km$)

$90° - \varphi'_1$ y $90° - \varphi'_2$ son arcos de la esfera de radio N_1

$\varphi'_1 = \varphi_1$

→ $P_1 P' P_2$

$$sen\varphi'_2 = sen\varphi_1 cos\widehat{P_1P_2} + cos\varphi_1 sen\widehat{P_1P_2}\, cos\alpha_1$$

$$\varphi'_2 = \varphi_1 + \Delta\varphi' \;\text{ y }\; \widehat{P_1P_2} = S_{12}/N_1$$

$$sen(\varphi_1 + \Delta\varphi') = sen\varphi_1 cos\frac{S_{12}}{N_1} + cos\varphi_1 sen\frac{S_{12}}{N_1} cos\alpha_1$$

Desarrollando en serie

$$\Delta\varphi' = \frac{S_{12}}{N_1}cos\alpha_1 - \frac{{S_{12}}^2}{2{N_1}^2}tg\varphi_1 - \frac{{S_{12}}^3}{6{N_1}^3}cos\alpha_1 + \frac{\Delta\varphi'^2}{2}tg\varphi_1 + \frac{\Delta\varphi'^3}{6}$$

$$\Delta\varphi' = \frac{S_{12}}{N_1}cos\alpha_1 - \frac{{S_{12}}^2}{2{N_1}^2}sen^2\alpha_1 tg\varphi_1 - \frac{{S_{12}}^3}{6{N_1}^3}cos\alpha_1 sen^2\alpha_1(1 + 3tg^2\varphi_1)$$

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Geodesia

Problema Geodésico Directo[7,9,10]: $\longrightarrow$ $\varphi_1, \lambda_1, \alpha_1, S_{12} \rightarrow \varphi_2, \lambda_2, \bar{\alpha}_2$

Fórmulas de Puissant

$$N_1 \Delta\varphi' = M_m \Delta\varphi$$

$$M_m = M_1 + \left.\frac{dM}{d\varphi}\right|_1 \frac{\Delta\varphi}{2} + \ \ldots \longrightarrow M_m = M_1 + \frac{3}{2}\frac{M_1 e^2 sen\varphi_1 cos\varphi_1}{(1 - e^2 sen^2\varphi_1)}\Delta\varphi$$

$$\Delta\varphi = \delta\varphi - A\,\delta\varphi\,\Delta\varphi \qquad A = \frac{3}{2}\frac{e^2 sen\varphi_1 cos\varphi_1}{(1 - e^2 sen^2\varphi_1)}$$

$$\delta\varphi = \frac{S_{12}}{M_1} cos\alpha_1 - \frac{{S_{12}}^2}{2N_1M_1} sen^2\alpha_1 tg\varphi_1 - \frac{{S_{12}}^3}{6{N_1}^2 M_1} cos\alpha_1 sen^2\alpha_1 (1 + 3tg^2\varphi_1)$$

$$\Delta\varphi \cong \delta\varphi \ \rightarrow \delta\varphi\Delta\varphi \cong (\delta\varphi)^2$$

$$\Delta\varphi = S_{12} cos\alpha_1 B - {S_{12}}^2 sen^2\alpha_1 C - h\,{S_{12}}^2 sen^2\alpha_1 D - (\delta\varphi)^2 A$$

$$B = \frac{1}{M_1} \qquad C = \frac{tg\varphi_1}{2N_1M_1} \qquad D = \frac{1 + 3tg^2\varphi_1}{6{N_1}^2} \qquad h = \frac{S_{12} cos\alpha_1}{M_1}$$

$$\boxed{\varphi_2 = \varphi_1 + \Delta\varphi}$$

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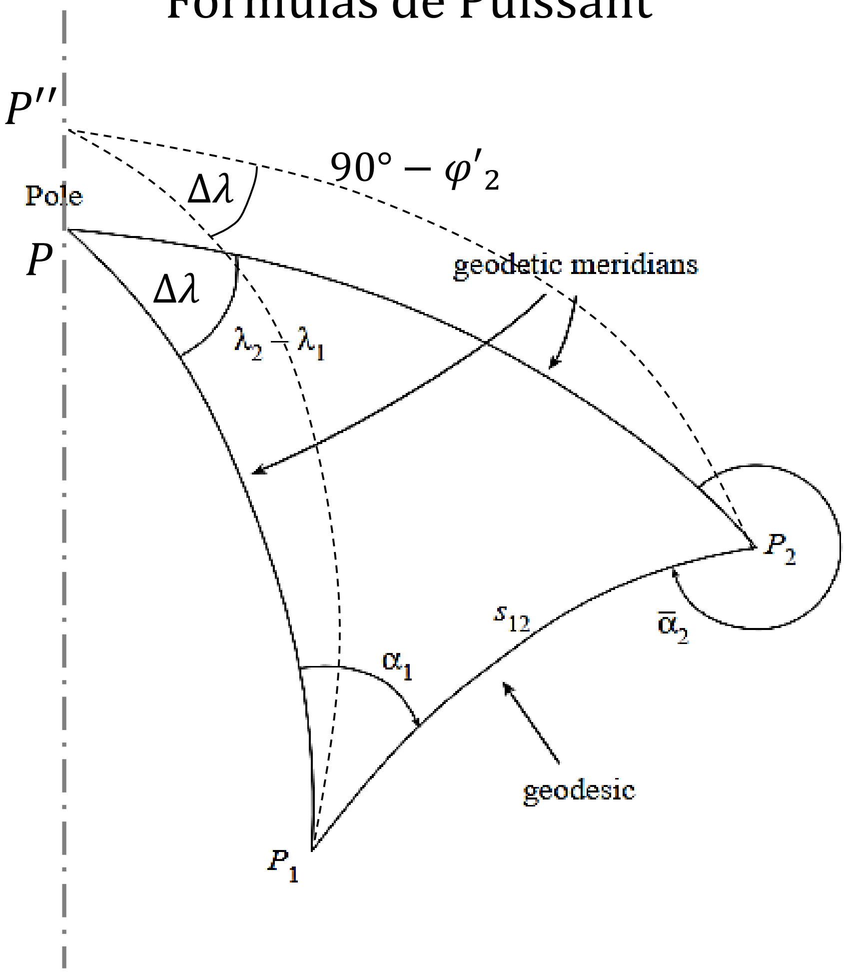
Fórmulas de Puissant



- Esfera de radio N_2, tangente en P_2
- Casi coincidente en P_1 (si $S < 100\ km$)
- $90° - \varphi'_2$ arco de la esfera de radio N_2
- $\varphi'_2 = \varphi_2$

$\Longrightarrow$ $P_1 P'' P_2$

$$\frac{sen\Delta\lambda}{sen\dfrac{S_{12}}{N_2}} = \frac{sen\alpha_1}{cos\varphi_2} \longrightarrow sen\Delta\lambda = sen\frac{S_{12}}{N_2}\frac{sen\alpha_1}{cos\varphi_2}$$

$$\Delta\lambda = \frac{S_{12}}{N_2} sen\alpha_1 sec\varphi_2 \left[1 - \frac{{S_{12}}^2}{6{N_2}^2}(1 - sen^2\alpha_1 sec^2\varphi_2)\right]$$

$$\boxed{\lambda_2 = \lambda_1 + \Delta\lambda}$$

(7) Rapp, Richard H. (1991). *Geometric geodesy part I*. Department of Geodetic Science and Surveying, Ohio State University.
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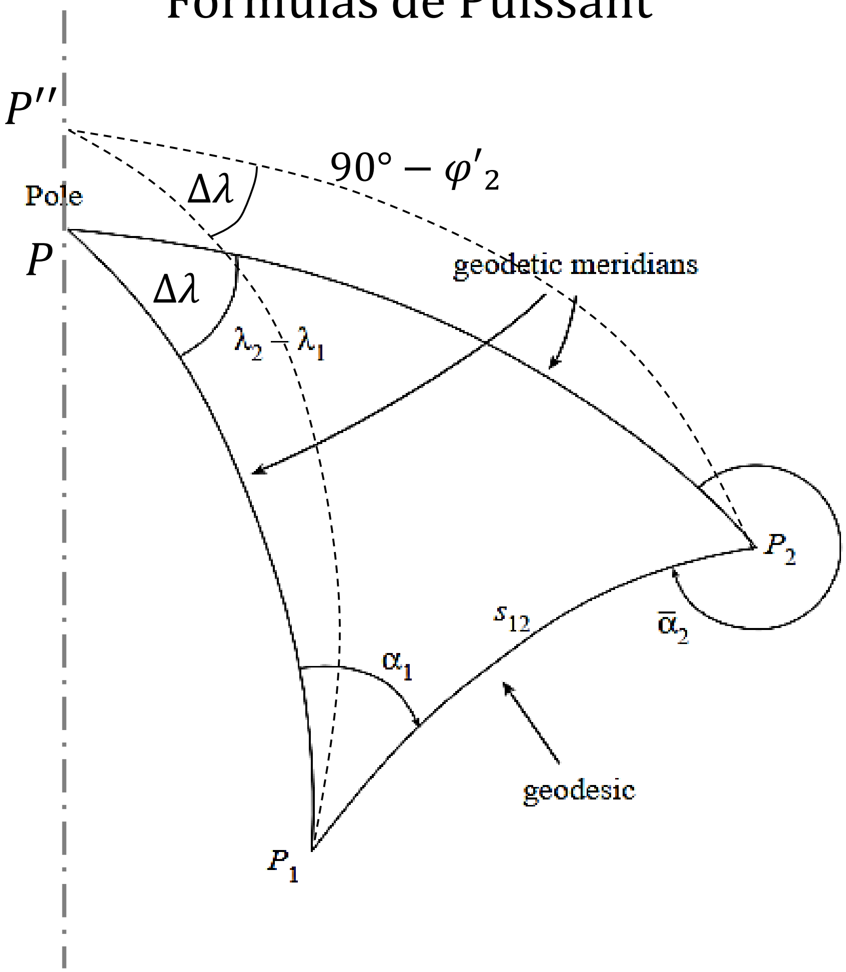
(10) Jordan, W. (1962). *Handbook of Geodesy* (Vol. 3). Corps of Engineers, United States Army, Army Map Service

Geodesia

Problema Geodésico Directo[7,9,10]: $\longrightarrow$ $\varphi_1, \lambda_1, \alpha_1, S_{12} \rightarrow \varphi_2, \lambda_2, \bar{\alpha}_2$

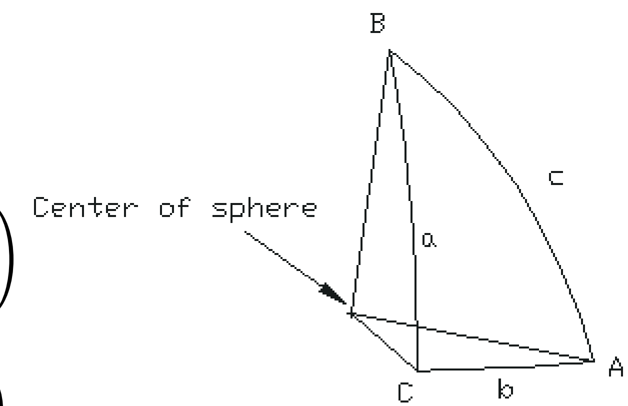
Fórmulas de Puissant



- Esfera de radio N_2, tangente en P_2
- Casi coincidente en P_1 (si $S < 100\ km$)
- $90° - \varphi'_2$ arco de la esfera de radio N_2
- $\varphi'_2 = \varphi_2$

$\Longrightarrow$ $P_1 P'' P_2$

$$tg\left(\frac{A+C}{2}\right) = \frac{cos\left(\frac{a-c}{2}\right)}{cos\left(\frac{a+c}{2}\right)} cotg\left(\frac{B}{2}\right)$$



$$tg\left(\frac{\alpha_1 + 360 - \bar{\alpha}_2}{2}\right) = \frac{cos\left(\frac{(90° - \varphi_2) - (90° - \varphi_1)}{2}\right)}{cos\left(\frac{(90° - \varphi_2) + (90° - \varphi_1)}{2}\right)} cotg\left(\frac{\Delta\lambda}{2}\right)$$

$$\bar{\alpha}_2 = \alpha_1 + \Delta\alpha \pm 180°$$

$$tg\left(\frac{\Delta\alpha}{2}\right) = \frac{sen\left(\frac{\varphi_1 + \varphi_2}{2}\right)}{cos\left(\frac{\Delta\varphi}{2}\right)} tg\left(\frac{\Delta\lambda}{2}\right) = \frac{sen\varphi_m}{cos\left(\frac{\Delta\varphi}{2}\right)} tg\left(\frac{\Delta\lambda}{2}\right)$$

$$\Delta\alpha = \Delta\lambda\ sen\varphi_m sec\frac{\Delta\varphi}{2} + \frac{\Delta\lambda^3}{12}\left(sen\varphi_m sec\frac{\Delta\varphi}{2} - sen^3\varphi_m sec^3\frac{\Delta\varphi}{2}\right)$$

$$\boxed{\bar{\alpha}_2 = \alpha_1 + \Delta\alpha \pm 180°}$$

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Geodesia

Problema Geodésico Inverso[7,9,10]: $\longrightarrow$ $\varphi_1, \lambda_1, \varphi_2, \lambda_2 \rightarrow \alpha_1, S_{12}, \bar{\alpha}_2$

Fórmulas de Puissant

$$\Delta\lambda = \frac{S_{12}}{N_2} sen\alpha_1 sec\varphi_2 \left[1 - \frac{{S_{12}}^2}{6{N_2}^2}(1 - sen^2\alpha_1 sec^2\varphi_2)\right]$$

$$S_{12}\ sen\alpha_1 = \frac{\Delta\lambda N_2\ cos\varphi_2}{\left[1 - \frac{{S_{12}}^2}{6{N_2}^2}(1 - sen^2\alpha_1 sec^2\varphi_2)\right]} \longrightarrow S_{12}\ sen\alpha_1 = \frac{\Delta\lambda N_2\ cos\varphi_2}{1}$$

$$\Delta\varphi = S_{12}cos\alpha_1 B - {S_{12}}^2 sen^2\alpha_1 C - h\, {S_{12}}^2 sen^2\alpha_1 D - (\delta\varphi)^2 A$$

$$S_{12}cos\alpha_1 = \frac{1}{B}\left[\Delta\varphi + {S_{12}}^2 sen^2\alpha_1 C + h\, {S_{12}}^2 sen^2\alpha_1 D + (\delta\varphi)^2 A\right] \longrightarrow S_{12}cos\alpha_1 = \frac{1}{B}\left[\Delta\varphi + {S_{12}}^2 sen^2\alpha_1 C\right]$$

$$tg{\alpha^0}_1 = \frac{S_{12}\ sen\alpha_1}{S_{12}cos\alpha_1}$$

$$S^0{}_{12} = \sqrt{(S_{12}\ sen\alpha_1)^2 + (S_{12}cos\alpha_1)^2}$$

$$h^0 = \frac{S^0{}_{12}cos{\alpha^0}_1}{M_1}$$

$$\Delta\alpha = \Delta\lambda\ sen\varphi_m sec\frac{\Delta\varphi}{2} + \frac{\Delta\lambda^3}{12}\left(sen\varphi_m sec\frac{\Delta\varphi}{2} - sen^3\varphi_m sec^3\frac{\Delta\varphi}{2}\right) \qquad \bar{\alpha}_2 = \alpha_1 + \Delta\alpha \pm 180°$$

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(9) Jekeli, C. (2006). *Geometric reference systems in geodesy*. Division of Geodetic Science, School of Earth Sciences. Ohio State University.
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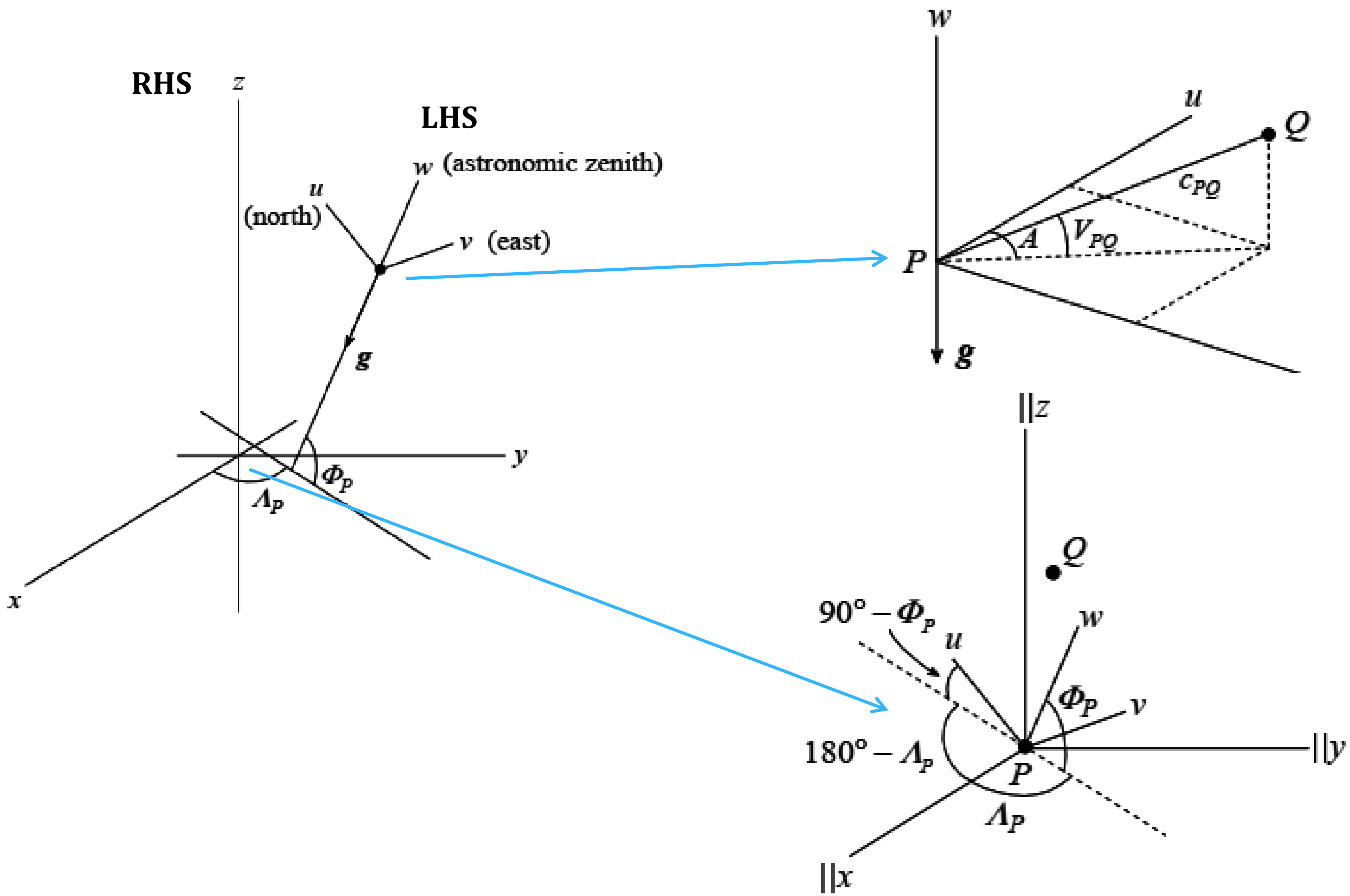
Transformación de coordenadas (globales ⇔ locales) [7,9,10]:



$$u_{PQ} = c_{PQ} \cos V_{PQ} \cos A_{PQ}$$
$$v_{PQ} = c_{PQ} \cos V_{PQ} \sin A_{PQ}$$
$$w_{PQ} = c_{PQ} \sin V_{PQ}$$

$$\| \, x_{PQ} \equiv \Delta x_{PQ} = x_Q - x_P$$
$$\| \, y_{PQ} \equiv \Delta y_{PQ} = y_Q - y_P$$
$$\| \, z_{PQ} \equiv \Delta z_{PQ} = z_Q - z_P$$

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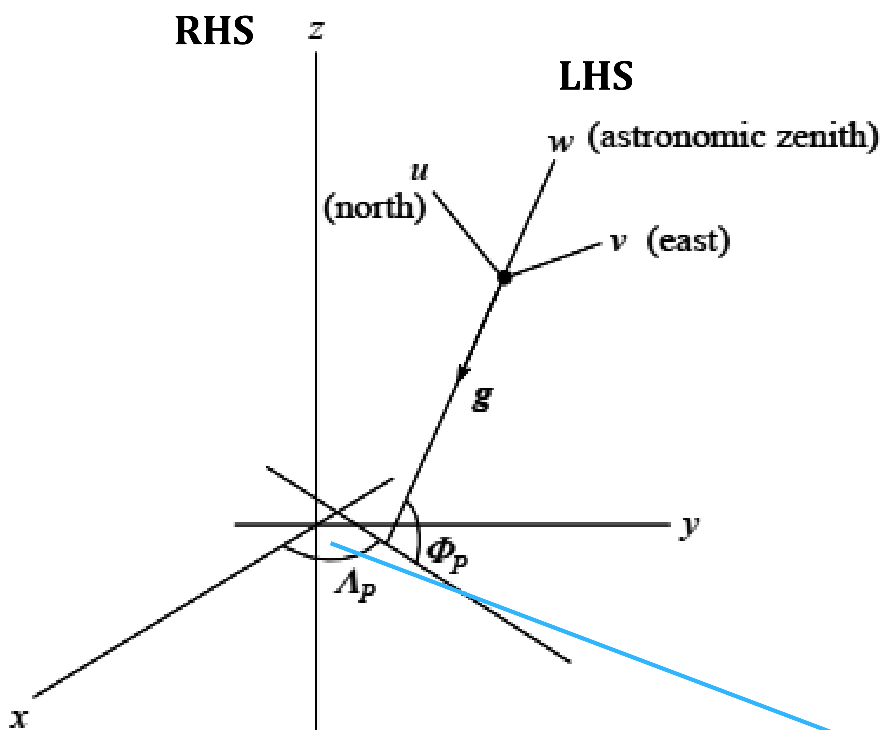
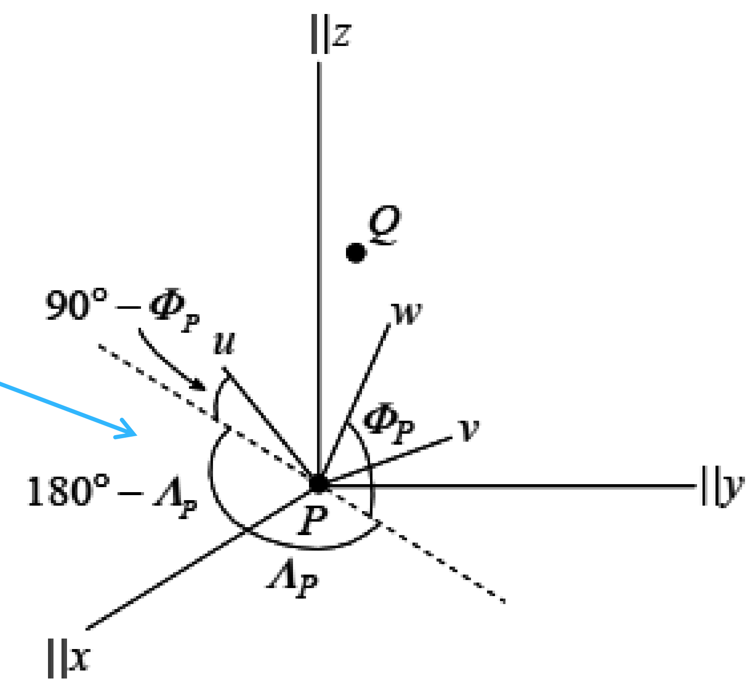


$$\begin{pmatrix}\Delta x_{PQ}\\ \Delta y_{PQ}\\ \Delta z_{PQ}\end{pmatrix} = R_3\left(180^\circ - \Lambda_P\right) R_2\left(90^\circ - \Phi_P\right)\begin{pmatrix}1 & 0 & 0\\ 0 & -1 & 0\\ 0 & 0 & 1\end{pmatrix}\begin{pmatrix}u_{PQ}\\ v_{PQ}\\ w_{PQ}\end{pmatrix},$$



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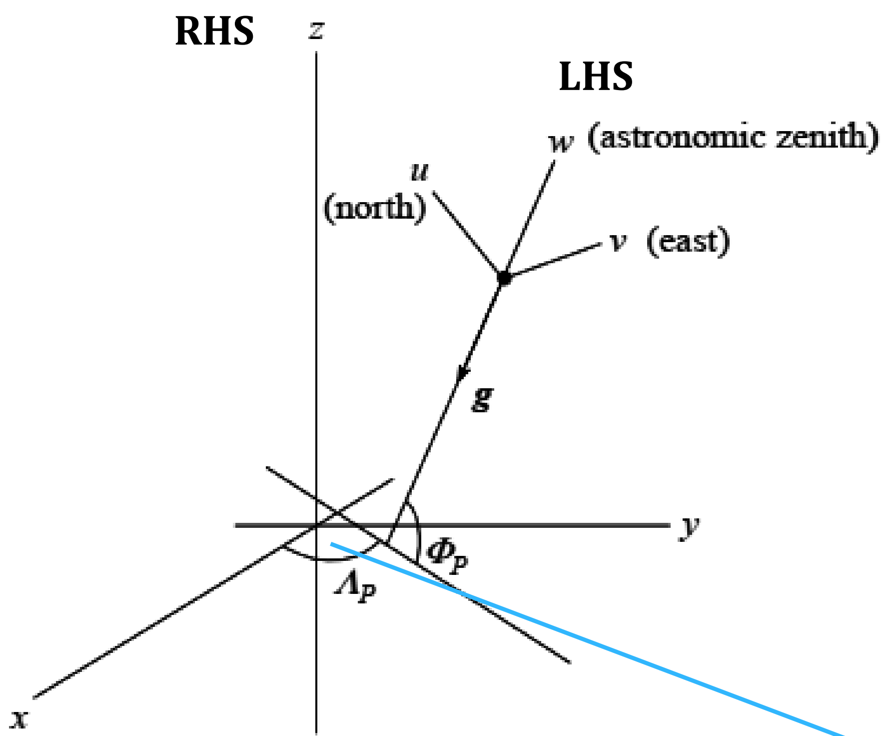
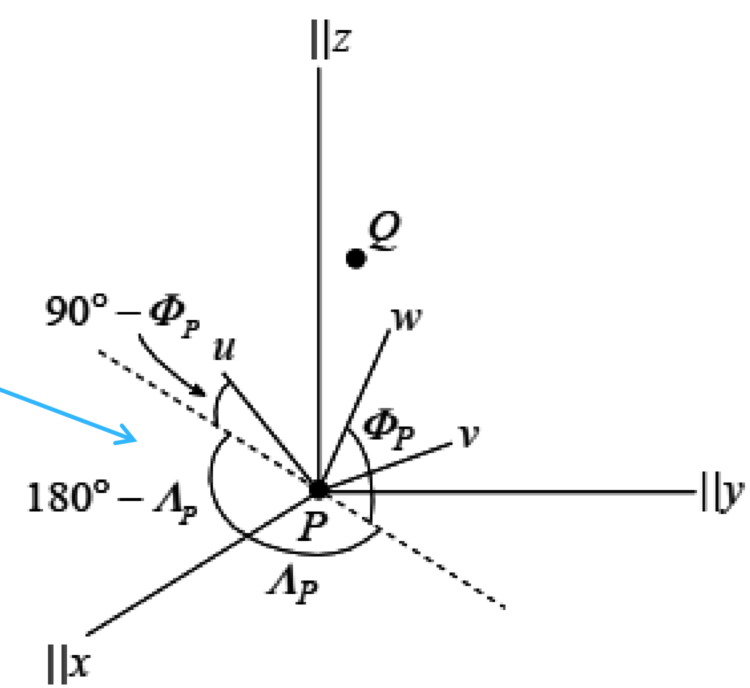


$$\begin{pmatrix} \Delta x_{PQ} \\ \Delta y_{PQ} \\ \Delta z_{PQ} \end{pmatrix} = \begin{pmatrix} -\sin\Phi_P \cos\Lambda_P & -\sin\Lambda_P & \cos\Phi_P \cos\Lambda_P \\ -\sin\Phi_P \sin\Lambda_P & \cos\Lambda_P & \cos\Phi_P \sin\Lambda_P \\ \cos\Phi_P & 0 & \sin\Phi_P \end{pmatrix} \begin{pmatrix} c_{PQ} \cos V_{PQ} \cos A_{PQ} \\ c_{PQ} \cos V_{PQ} \sin A_{PQ} \\ c_{PQ} \sin V_{PQ} \end{pmatrix}$$



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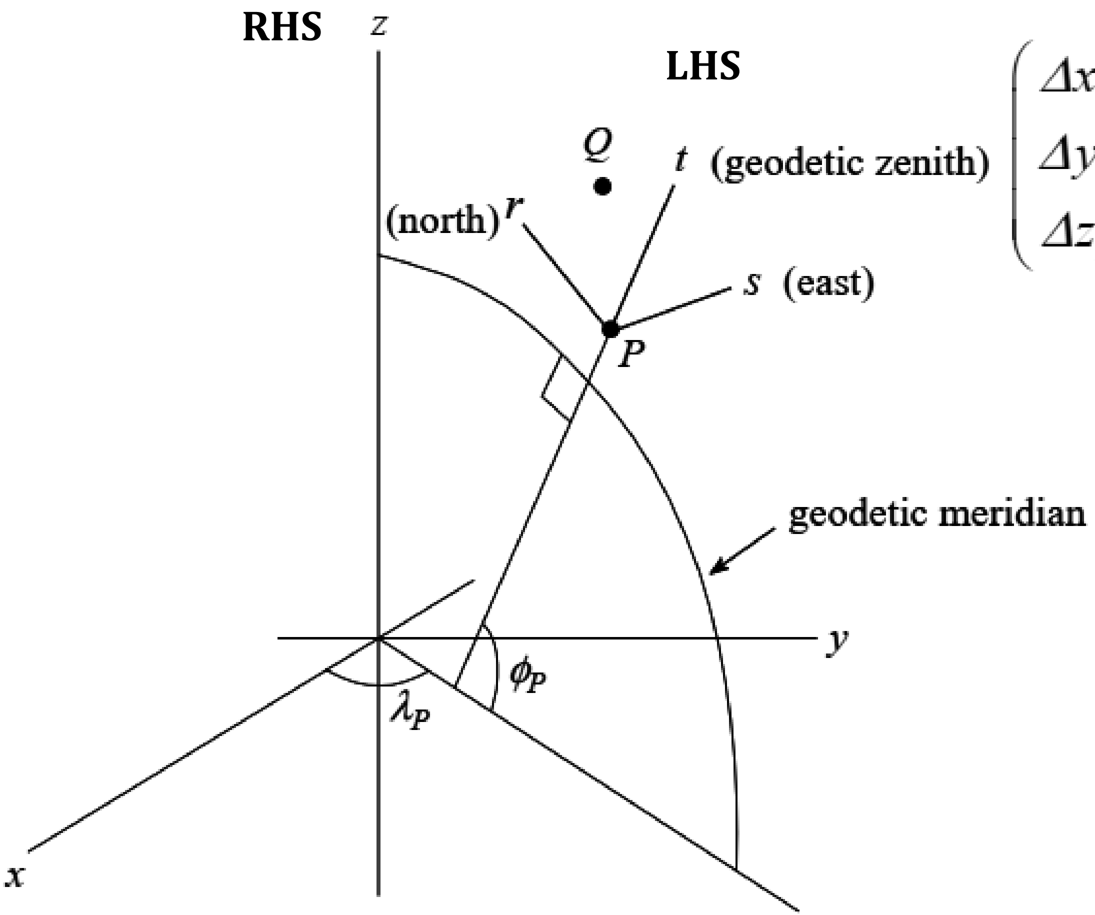
Transformación de coordenadas (globales ⇔ locales) [7,9,10]:



$$\begin{pmatrix} \Delta x_{PQ} \\ \Delta y_{PQ} \\ \Delta z_{PQ} \end{pmatrix} = \begin{pmatrix} -\sin\phi_P \cos\lambda_P & -\sin\lambda_P & \cos\phi_P \cos\lambda_P \\ -\sin\phi_P \sin\lambda_P & \cos\lambda_P & \cos\phi_P \sin\lambda_P \\ \cos\phi_P & 0 & \sin\phi_P \end{pmatrix} \begin{pmatrix} c_{PQ} \cos v_{PQ} \cos\alpha_{PQ} \\ c_{PQ} \cos v_{PQ} \sin\alpha_{PQ} \\ c_{PQ} \sin v_{PQ} \end{pmatrix}$$

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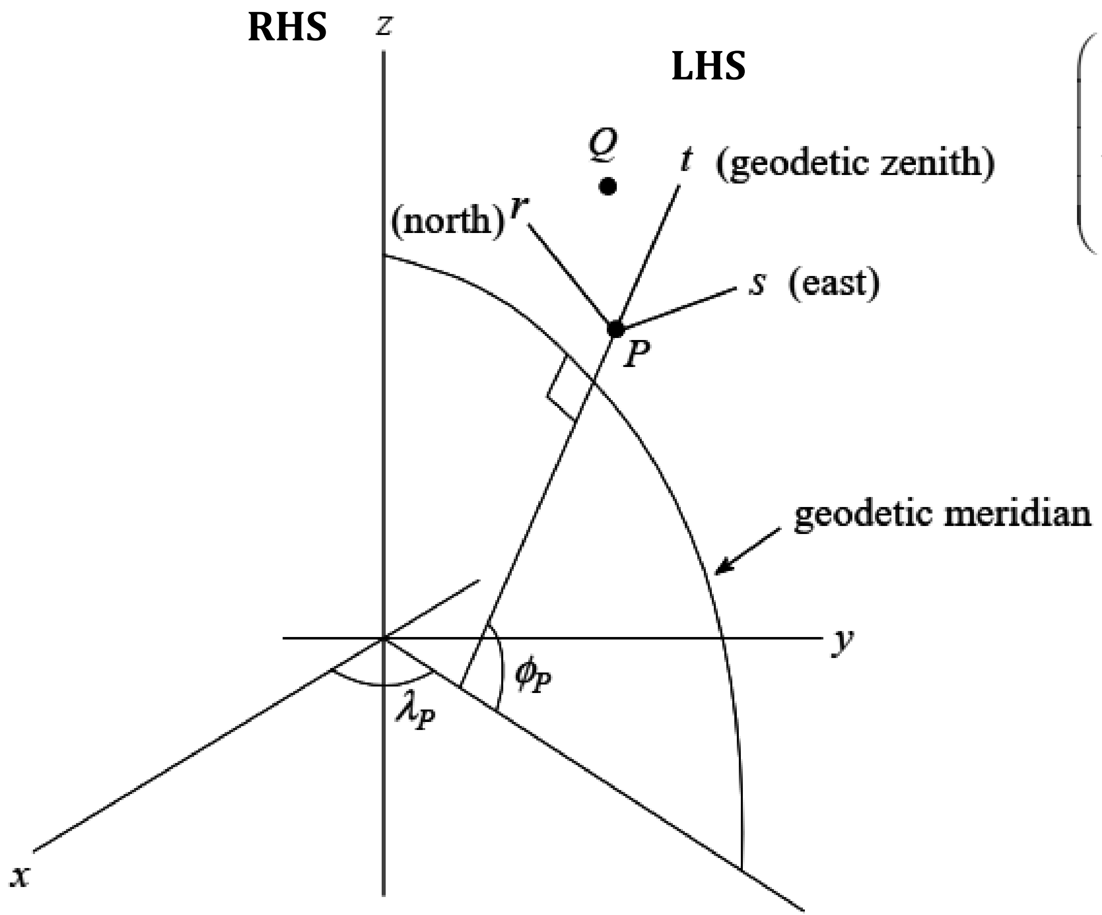
Transformación de coordenadas (globales ⇔ locales) [7,9,10]:



$$\begin{pmatrix} \Delta x_{PQ} \\ \Delta y_{PQ} \\ \Delta z_{PQ} \end{pmatrix} = \begin{pmatrix} -\sin\phi_P \cos\lambda_P & -\sin\lambda_P & \cos\phi_P \cos\lambda_P \\ -\sin\phi_P \sin\lambda_P & \cos\lambda_P & \cos\phi_P \sin\lambda_P \\ \cos\phi_P & 0 & \sin\phi_P \end{pmatrix} \begin{pmatrix} c_{PQ} \cos v_{PQ} \cos\alpha_{PQ} \\ c_{PQ} \cos v_{PQ} \sin\alpha_{PQ} \\ c_{PQ} \sin v_{PQ} \end{pmatrix}$$

$$\tan\alpha_{PQ} = \frac{-\Delta x_{PQ} \sin\lambda_P + \Delta y_{PQ} \cos\lambda_P}{-\Delta x_{PQ} \sin\phi_P \cos\lambda_P - \Delta y_{PQ} \sin\phi_P \sin\lambda_P + \Delta z_{PQ} \cos\phi_P},$$

$$\sin v_{PQ} = \frac{1}{c_{PQ}} \left(\Delta x_{PQ} \cos\phi_P \cos\lambda_P + \Delta y_{PQ} \cos\phi_P \sin\lambda_P + \Delta z_{PQ} \sin\phi_P \right),$$

$$c_{PQ} = \sqrt{\Delta x_{PQ}^2 + \Delta y_{PQ}^2 + \Delta z_{PQ}^2} \,.$$

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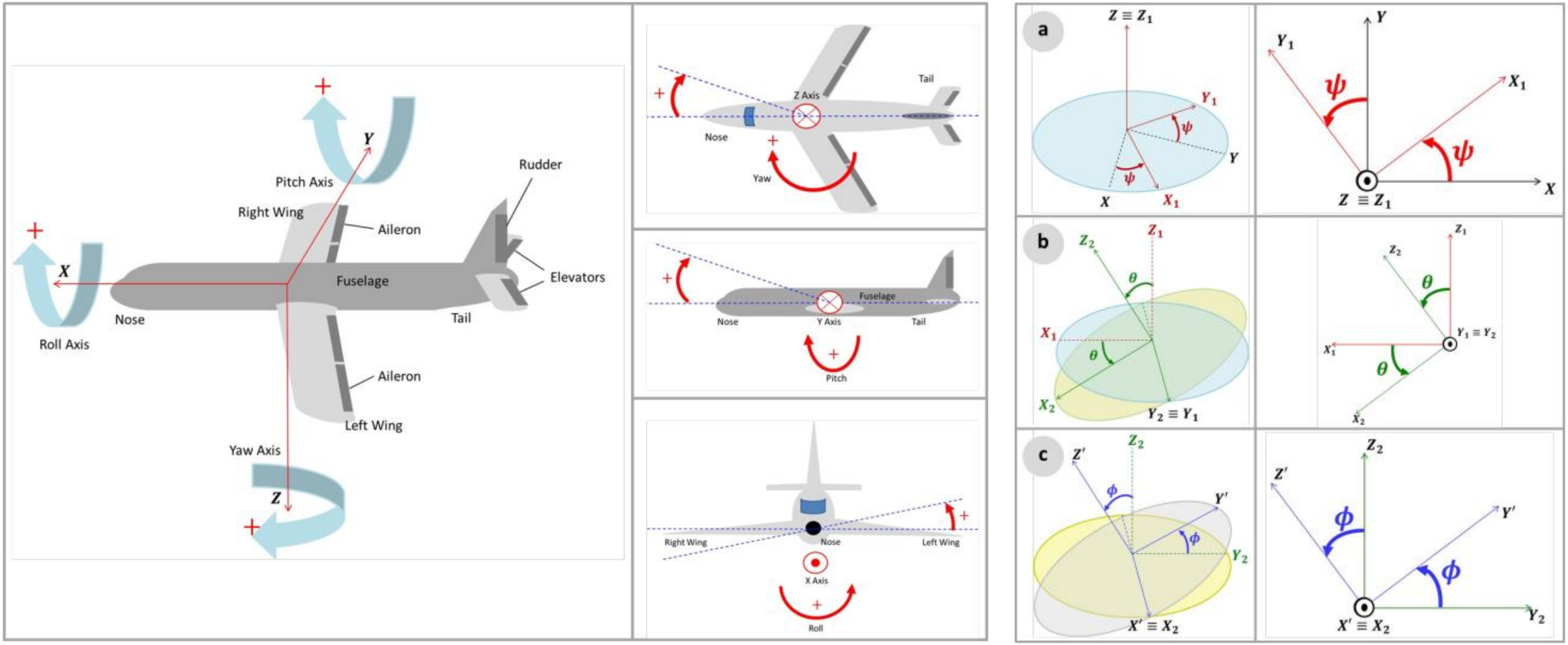

Fuente: Correia, C.A.M.; Andrade, F.A.A.; Sivertsen, A.; Guedes, I.P.; Pinto, M.F.; Manhães, A.G.; Haddad, D.B. Comprehensive Direct Georeferencing of Aerial Images for Unmanned Aerial Systems Applications. *Sensors* **2022**, *22*, 604.

(7) Rapp, Richard H. (1991). ***Geometric geodesy part I*****. Department of Geodetic Science and Surveying, Ohio State University.**
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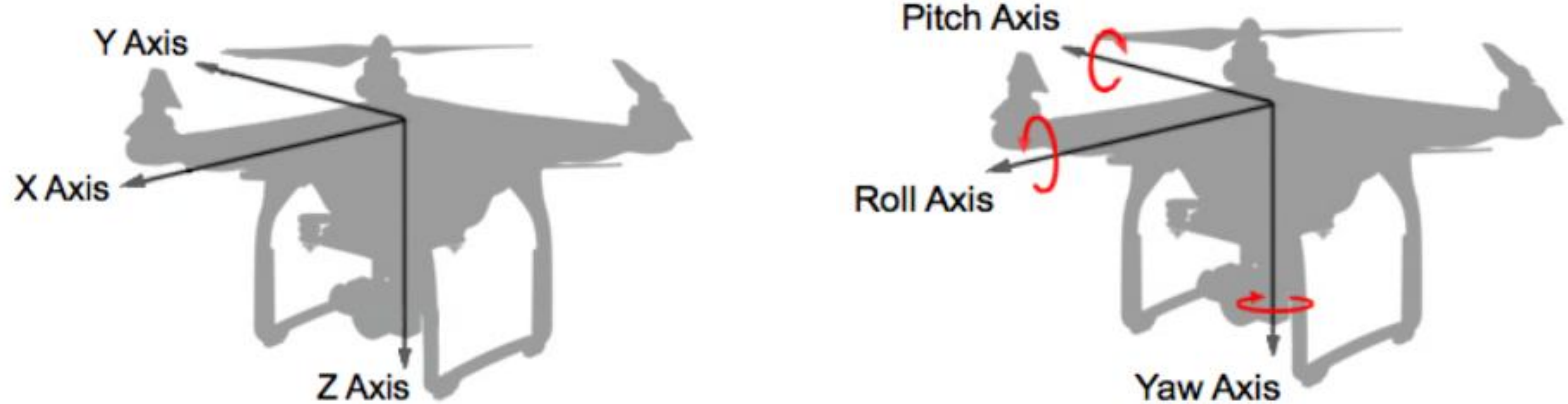
Coordinate Systems

Description of aircraft movement is dependent on the location and orientation of coordinate axes that make a coordinate system (or frame of reference). Many coordinate systems exist, but the two used in the DJI Mobile SDK are relative to the aircraft body (body frame), and relative to the ground (world frame).

Body Coordinate System

The body coordinate system is relative to the aircraft itself. Three perpendicular axes are defined such that the origin is the center of mass, and the **X** axis is directed through the front of the aircraft and the **Y** axis through the right of the aircraft. Using the coordinate right hand rule, the **Z** axis is then through the bottom of the aircraft.



Fuente: https://developer.dji.com/mobile-sdk/documentation/introduction/flightController_concepts.html

(7) Rapp, Richard H. (1991). *Geometric geodesy part I*. Department of Geodetic Science and Surveying, Ohio State University.
(9) Jekeli, C. (2006). *Geometric reference systems in geodesy*. Division of Geodetic Science, School of Earth Sciences. Ohio State University.
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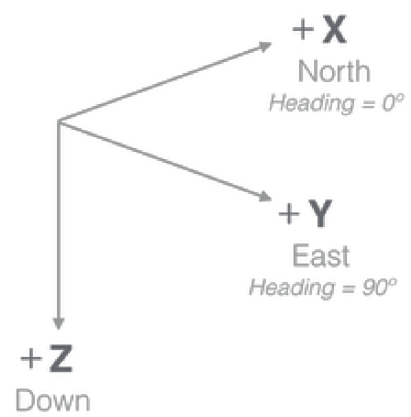
Ground (World) Coordinate System

A popular ground or world coordinate system used for aircraft aligns the positive **X**, **Y** and **Z** axes with the directions of **North**, **East** and **down**. This convention is called **North-East-Down** or **NED**.

Positive Z pointing downward can take some getting used to, but it is convenient as X and Y are then consistent with the right hand rule and normal navigation heading angles. A heading angle of 0° will point toward the North, and +90° toward the East.

The origin for a NED coordinate system is usually a point in the world frame (like take-off location).



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